Introduction

We reconsider competitive general equilibrium theory with constant returns as a framework for studying technological innovation and its effect upon growth. As in Aghion and Howitt [1992], Grossman and Helpman [1991] and Romer [1990] new goods and new technologies are introduced because of the role of individual entrepreneurs in seeking out profitable opportunities. Unlike those models we do not assume monopolistic competition or increasing returns to scale: the technology set faced by our economic agents is a convex cone and competitive equilibria are efficient. We suppose there are a countably infinite number of produceable commodities. Technological progress takes place because entrepreneurs find it advantageous to introduce new activities that produce new commodities, and these new commodities themselves make profitable the employment of other activities that make use of them. Although, in the ensuing equilibrium, entrepreneurs do not actually end up with a profit, it is their pursuit of profit that drives innovation. The result is an abstract, dynamic model in the spirit of Schumpeter's *Theory of Economic Development* in which relative prices are altered by the action of entrepreneurs and economic growth takes place in a fluctuating as opposed to a balanced way. Despite the existence of infinitely many commodities and activities, efficient allocations can be decentralized in the classical way using the first and second welfare theorems, and a transversality condition.

At any point in time a finite number of goods are produced through a finite number of activities. Barring adoption of new goods or activities, this defines a neoclassical production economy in the style of McKenzie [1981, 1986], the most popular example of which is the AK model of Jones and Manuelli [1990] and Rebelo [1991]. The AK model has a simple dynamics: convergence to the balanced growth path ocurrs in one period, after which the economy grows at a constant rate. Modulo technical complications, a similar result holds true for more general, multisector versions of the AK model such as those of Bewley [1982], McKenzie [1995] and Yano [1984] in which growth at a constant rate is driven by accumulation of the capital. In our framework we can meaningfully distinguish between the *two main sources* of economic growth: (i) capital accumulation; (ii) adoption of more efficient techniques and new goods. The first is a well understood property of economies with linear technologies, to which we have nothing additional to contribute. Contrary to previous models, though, in ours adoption of more efficient techniques and of new goods is continuously undertaken by entrepreneurs searching for profitable opportunities. The article concentrates on this largely unexplored feature of the theory: endogenous technological innovation under perfect competition.

The most elementary example of our theory is a simple vintage capital formulation of the AK model, which we discuss in Section 2. This exhibits the two basic sources of growth mentioned above. However, this type of model does not allow for the study of important issues associated with innovative activity. Three stand out in particular: (1) the relation between the social value of an innovation and the gains accruing to the innovator; (2) the pricing of "ideas" or, more generally, of goods for which the initial set up cost is large compared to the marginal cost of reproduction; and, (3) long run dependence upon initial conditions, or what we might describe as "growth anomalies": that an initially poorer country may in the long-run wind up using a superior technology, or the possibility that a modest increase in savings may lead to a dramatic increase in growth. Models of technology adoption with fixed costs or other increasing returns, in which some economic agents enjoy monopoly power, provide clearcut answers to these problems. Previous models of growth under constant returns are unable even to formulate them. In this sense, our model is both a contribution to dynamic competitive theory, and an alternative tool to increasing returns and monopoly power for modelling economic innovation.

The setting we examine has a continuum of infinitely lived identical households. They derive utility from being able to enjoy a (possibly ever-increasing) amount of "characteristics", as in

Lancaster [1966] and Stokey [1988]. There are a finite number of such characteristics and each commodity is identified with a vector of them. So while the number of potential commodities is infinite and viable ones change from period to period, the number of characteristics they produce is finite and time-invariant. Our utility functions are standard, additively separable utility functions, with a period utility a function on a fixed finite dimensional space. In the spirit of von Neumann we study an environment with an activity analysis technology. However, as for produceable commodities, the number of potential activities is allowed to be countably infinite. An activity is characterized by a pair of input and output vectors and displays constant returns to scale. The input goods used in production come from output in the previous period. The level at which an activity is operated is limited by the availability of inputs and by aggregate demand and, therefore, relative prices. It is well known that arbitrary diminishing returns technologies can be approximated by activity analysis technologies. In addition, activities have a convenient interpretation as “inventions”, “blueprints” or “ideas” and provide a convenient way of modelling changes in the production possibilities set over time.

We concentrate on the factors determining the adoption of new activities. We do this because we share the view that “it is entirely immaterial whether an innovation implies scientific novelty or not. Although most innovations can be traced to some conquest in the realm of either theoretical or practical knowledge, there are many which cannot. Innovation is possible without anything we should identify as invention and invention does not necessarily induce innovation, but produces of itself no economically relevant effect at all. The economic phenomena which we observe in the special case in which innovation and invention coincide do not differ from those we observe in cases in which preexisting knowledge is made use of.” (Schumpeter [1939, III.A]).

The remaining of the paper is organized as follows. Section 2 introduces the abstract theory and illustrate its operation by means of the vintage/AK model. Section 3 decentralizes the optimal allocation, proves the two welfare theorems and show, by means of examples, how entrepreneurs compete and receive their reward. In so doing we also show how new goods, ideas in particular, are priced and how this may affect the distribution of income. Section 4 concentrates on growth anomalies and show that joint production can play the same role that fixed costs and externalities play in other models. Section 5 concludes.

The Model

Households

We consider an infinite horizon economy, $t = 0, 1, 2, \ldots$ with a continuum of homogeneous consumers. Consumers value characteristics $c_t \in \Re_+^J$ where J is the number of characteristics. The *period utility* provided by the consumption of characteristics during an interval of time, is denoted $u(c_t)$.

***Assumption* 1** The period utility $u(\bullet)$ is strictly increasing, concave, smooth, and bounded below.

Total *lifetime utility* is given by $U(c) = \sum_{t=1}^{\infty} \delta^{t-1} u(c_t)$, where $0 \leq \delta < 1$ is the common subjective discount factor.

The assumptions that the utility function is strictly increasing and concave are standard. The smoothness of the period utility function is convenient and, for a concave function, not terribly restrictive. The assumption that the period utility function is bounded below is technically useful. It insures that $U(c)$ is well defined (although possibly infinite). As we are concerned with the theory of growth, not the theory of subsistence, we are primarily interested in the behavior of $u(\bullet)$ for large and possibly growing quantities of consumption, so the behavior of the utility function near $c = 0$ is rather secondary to our ends. Moreover, from an intellectual perspective, if $u(0) = -\infty$ this has the counterfactual implication that no amount of consumption, however large, will compensate for any probability of $c = 0$, no matter how small this probability might be. footnote

Characteristics c_t are acquired through the consumption of commodities. The potential number of commodities is countably infinite although, at each point in time, only a finite number of them will be produced or consumed. The period commodity space consists of the set $X \subseteq \ell_+^\infty$ composed of sequences $(x_1, x_2, \ldots, x_n, \ldots) \geq 0$ for which $x_n = 0$ for all but finitely many n. The overall commodity space is then

$$\tilde{X} = \times_{t=0}^{\infty} X$$

The vector of characteristics acquired by the consumption of a single unit of commodity n is denoted by $C_n \in \Re_+^J$. These induce a linear map $C : X \to \Re_+^J$. So, if x_t denotes the vector of commodities consumed at time t the characteristics enjoyed by the agent are $c_t = Cx_t$. We also denote by C^j the map $C^j : X \to \Re_+$ from commodity vectors to the amount of characteristic $j = 1, \ldots, J$ acquired. footnote

Production

Production takes place through linear activities. An activity a is a pair of vectors $(k(a); y(a))$ where $k(a) \in X$ denotes the input of commodities entering the activity at the end of period t, and $y(a) \in X$ the output of commodities made available by the activity at the beginning of the following period. During period $t + 1$ the outputs can either be consumed or used as inputs for further production.

The set of potential activities is countable and denoted by $\mathcal{A}$.

***Assumption* 2** $\mathcal{A} \neq \emptyset$.

Any collection of activities $A \subseteq \mathcal{A}$ can be simultaneously operated at every non negative level $\lambda(a)$ as long as inputs $\sum_{a \in A} \lambda(a)k(a)$ are available. We assume also that $\mathcal{A}$ satisfies *no-free-lunch* and it allows for free disposal. footnote

***Assumption* 3** For all $a \in \mathcal{A}$ if $k(a) = 0$, then $y(a) = 0$; if $(k(a), y(a))$ is an activity in $\mathcal{A}$ with $y(a) \neq 0$, then the activity $(k(a); 0)$ is also in $\mathcal{A}$.

Denote the vector of activity levels at time t as $\lambda_t \in \Re_+^{\mathcal{A}}$ and define the aggregate stock of capital at time t as $k_t = \sum_{a \in \mathcal{A}} \lambda_{t-1}(a)y(a) - x_t$, where x_t is aggregate consumption at time t.

Definition *A pair $\lambda \in (\times_{t=0}^{\infty} \Re_+^{\mathcal{A}}), k \in \tilde{X}$ is called a* ***production plan*** *and $x \in \tilde{X}$ is called a* ***consumption plan****. Together they determine an* ***allocation****.*

Definition *The allocation $\lambda \in (\times_{t=0}^{\infty} \Re_+^{\mathcal{A}}), k \in \tilde{X}, x \in \tilde{X}$ is a* ***feasible*** *allocation for the* ***initial condition*** *k_0 if*

$$\sum_{a \in \mathcal{A}} \lambda_t(a)y(a) \geq k_{t+1} + x_{t+1}$$

$$k_t \geq \sum_{a \in \mathcal{A}} \lambda_t(a)k(a),$$

for all $t = 0, 1, \ldots$

We call an activity $a \in \mathcal{A}$ *viable* at t for initial condition k_0, if there exists a socially feasible allocation starting from k_0 and such that $\lambda_t(a) > 0$. We denote the set of viable activities at time t from k_0 by $A_t(k_0)$. Note that $A_t(k_0)$ will, in general, be a proper subset of $\mathcal{A}$. This occurs whenever, for some $a \in \mathcal{A}$ and some index n denoting a commodity, we have $k_n(a) > 0$ and there is no feasible allocation from k_0 such that the vector of outputs y_{t-1} has $y_{n,t-1} > 0$. Similarly, we call a commodity n *viable* at time t for the initial condition k_0 if there exists a socially feasible allocation starting from

k_0 and such that $y_{n,t} > 0$. Our analysis is greatly simplified by assuming that, for any initial vector of capital stocks k_0, the set of viable activities is finite.

Assumption **4** For all $k_0 \in X$, $A_t(k_0)$ is finite.

Finally, we assume that it is possible to produce all characteristics in every period.

Assumption **5** For all k_0 there exists a feasible allocation with $C^j x_t > 0$ for all $1 \leq j \leq J$ and $t > 0$.

Example – Vintage Capital and AK Models

We illustrate the model with a simple class of economies. There is a single Characteristic, so that $J = 1$. The period utility function has the CES form $u(c) = -(1/\theta)[c]^{-\theta}$, $\theta > 0$. There are two types of commodities, a single consumption good and an infinite sequence of different vintages of capital, indexed by $i = 0, 1, \ldots$. The consumption good may be converted into the desired characteristic on a 1–1 basis. We write a commodity vector $x = (z, \kappa)$ where z is a scalar denoting the consumption good and κ is an infinite vector of capital stocks of different vintages, and we let the symbol χ_i denote the vector consisting of one unit of vintage i capital and zero units of all other vintages. So, for example, $(0, \chi_2)$ is a commodity vector with 0 units of consumption, 1 unit of vintage 2 capital and zero units of everything else.

There are 2 sequences of activities. One sequence of activities, $[0, \chi_i; \gamma^i, 0]$ with $\gamma > 1$, produces consumption from vintage i capital. Notice that different vintages of capital differ in how effective they are at producing consumption. The second sequence of activities, $[0, \chi_i; 0, \rho\chi_{i+1}]$ with $\rho > 0$, produces ρ units of vintage $i + 1$ capital from 1 unit of vintage i capital.

The endowment k_0 is a single unit of vintage 0 capital. Notice that in this setup, at time t there can only be capital of vintage t, so that this is a vintage capital model, as in Solow [1960], Benhabib and Rustichini [1991] or Chari and Hopenhayn [1991].

We look for a social optimum where capital grows exponentially from one vintage to the next, and a constant fraction ϕ of the stock of current vintage capital is used in the production of the consumption good. Let κ_{tt} denote the amount of vintage t capital at time t. One unit of this capital can be used to produce γ^t units of consumption good, yielding a marginal present value utility of $\delta^t \gamma^t (\phi\gamma^t \kappa_{tt})^{-\theta-1}$. Alternatively it can be used to produce ρ units of vintage $t + 1$ capital for time $t + 1$, leading to a present value marginal utility of $\rho\delta^{t+1}\gamma^{t+1}(\phi\gamma^{t+1}\kappa_{t+1,t+1})^{-\theta-1}$, where $\kappa_{t+1,t+1} = \rho(1 - \phi)\kappa_{tt}$. Equating payoffs from the two alternative uses of capital we get

$$\rho(1 - \phi) = (\delta\rho\gamma^{-\theta})^{1/(1+\theta)}$$

Notice that $\rho(1 - \phi)$ is the growth rate of the capital stock. The corresponding growth rate of consumption is

$$g_c = \gamma\rho(1 - \phi) = (\delta\rho\gamma)^{1/(1+\theta)}$$

If $\delta(\delta\rho\gamma)^{-\theta/(1+\theta)} \geq 1$ this yields an infinite present value of utility; otherwise total lifetime utility is finite, which is the interesting case.

Notice that while this example is based upon the most primitive form of technological innovation, in which technology improves in each period and at a rate that is exogenously fixed, the growth rate of the economy is endogenous, since it depends on the rate of capital accumulation. In particular, if the economy becomes more productive (as measured by ρ or γ), or more patient (as measured by δ or θ^{-1}), it grows faster.

It is easy to modify this example to endogenize the technology choice. If we introduce the activity $[0, \chi_i; 0, \beta\chi_i]$, then it becomes possible to reproduce the current vintage of capital. Now there is a choice: reproduce the existing vintage of capital and remain technologically stagnant, or move on to the next vintage of capital?

If $\beta > 1$ growth, in the sense of an unbounded accumulation of physical capital and ever growing flows of output, is feasible even in the absence of technological innovation. Assume, moreover, that $\beta > \rho$ so that a unit of current capital is more productive at reproducing itself than at producing the next quality. In other words, technological innovations are costly. Nevertheless, it is easily seen that when $\gamma\rho > \beta$ the new technology is sufficiently productive that the only vintage of capital produced is the latest possible one.

This example shows how to distinguish between the *two main sources* of economic growth (Schumpeter [1911, 1934]): (i) unbounded accumulation of reproducible inputs, due to constant returns to scale; (ii) adoption of more efficient methods of production, as embodied in new activities or goods. In this case, when $\gamma\rho > \beta$ the choice between unbounded accumulation and introduction of a new, superior machine is solved once and for all in favor of the second option.

Decentralization

The model presented so far, is one of optimal growth. Much of our interest, though, is in the role played by individual entrepreneurs in inducing technological change. As usual, these two are tied together by the welfare theorems describing how optimal allocations may be decentralized by competitive pricing schemes. After proving the relevant welfare theorems for this model, we examine the role of profits, competition and entrepreneurship in the introduction of new technologies and commodities.

Optimality and Supporting Prices

Definition *The allocation* $a^* = \left\{\lambda^* \in (\times_{t=1}^{\infty} \Re_+^{\mathcal{A}}),\ k^* \in \tilde{X},\ x^* \in \tilde{X}\right\}$ ***solves the social planner problem*** *for initial condition* k_0 *if it solves*

$$\max_{\lambda,k,x}\ U(c)$$

subject to: $c_t = Cx_t$, *and feasibility of the production plan.*

Let $p_t \in \Re_+^{\infty}$ be the price of commodities at time t and $p \in \mathcal{P} = (\times_{t=0}^{\infty} \Re_+^{\infty})$ a whole sequence of prices from time zero to infinity. In a competitive equilibrium these prices must satisfy two conditions: they should yield zero profits and support the preferences.

Definition *Given* $k \in \tilde{X}$ *and* $\lambda \in (\times_{t=0}^{\infty} \Re_+^{\mathcal{A}})$ *the prices* $p \in \mathcal{P}$ *satisfy the* ***zero profit condition*** *if,*

$$p_{t+1}y(a) - p_t k(a) \leq 0, \forall a \in A_t(k_0),\ t = 0,1,\ldots$$

with equality if $\lambda_t(a) > 0$. *That is profits*

$$\pi_{t+1}(a) = \lambda_t(a)[p_{t+1}y(a) - p_t k(a)] = 0$$

Definition *Given prices* $p \in \mathcal{P}$, *the sequence* $x^* \in \tilde{X}$ ***solves the consumer's maximization problem*** *if* x^* *is the argmax of*

$$\max\ U(c)$$

$$\text{subject to}: \quad c_t = Cx_t,\ \ \sum_{t=1}^{\infty} p_t x_t \leq \sum_{t=1}^{\infty} p_t x_t^*$$

Definition *The pair* $p \in \mathcal{P}$ *and* $x^* \in \tilde{X}$ *satisfy the* ***first order conditions*** *for consumer's maximization if*

$$p_{nt}^* \geq \delta^{t-1} Du(Cx_t^*) C_n$$

with equality unless $x^*_{nt} = 0$.

Definition *The pair* $p \in \mathcal{P}$ *and* $k^* \in X$ *satisfy the* ***transversality condition*** *if*

$$\lim_{T \to \infty} p_T k^*_T = 0$$

Definition *The feasible allocation* $\lambda \in (\times_{t=0}^{\infty} \Re_+^{\mathcal{A}}), k \in \tilde{X}, x \in \tilde{X}$ *and the price sequence* $p \in \mathcal{P}$ *are a* ***competitive equilibrium*** *if they satisfy the zero profits condition and solve the consumer's maximization problem.*

Decentralization Theorem

We give a statement of the first and second welfare theorems that fit our purpose. A proof can be found in the Appendix.

Theorem *Suppose assumptions* 1, 2, 3, 4 *and* 5 *hold. Suppose that* λ^*, k^*, x^* *is a feasible allocation given* k_0 *and that* $\sum_{t=1}^{\infty} \delta^{t-1} u(Cx^*_t) < \infty$. *Then the following three conditions are equivalent*:

(1) λ^*, k^*, x^* *solve the planner's problem for initial condition* k_0.

(2) *There exist prices* p^* *satisfying the zero profit condition and such that* x^* *solves the consumer maximization problem given* p^* *with* $\sum_{t=1}^{\infty} p^*_t x^*_t < \infty$.

(3) *There exist prices* p^* *satisfying the zero profit condition such that the pair* p^* *and* x^* *satisfies the first order conditions and the pair* p^* *and* k^* *satisfies the transversality condition.*

Entrepreneurship, Profits and Competition

Almost always, general equilibrium discussions of product innovation begins with some type of market imperfection, such as monopolistic competition, increasing returns or externalities. Our theory of innovation abstracts from these imperfections: entrepreneurs have well defined property rights to the full proceeds from their innovations; individual production processes display constant returns; there are no fixed costs and no unpriced spillover effects from innovation. Does this lead to an interesting theory of innovation? We believe it leads to a theory that, while more parsimonious than established ones, is more versatile and has at least the same amount of explanatory power. Although the basic ingredients of fixed factors, rents and sunk costs are familiar from the standard theory of competitive equilibrium, the way in which they fit together in an environment of growth and innovation is apparently not well understood. A review will serve to explain our claim that traditional theory can go a long way toward explaining endogenous technological innovation and entrepreneurial activity.

Consider a single entrepreneur who is contemplating introducing a new activity, either to produce an existing good more efficiently or to produce a brand new good. He anticipates the prices at which he will be able to buy inputs and sell his output, and introduces the innovation if, at those prices, he can command a premium over alternative uses of his endowment. He owns the rights to his innovation, meaning that he expects to be able to collect the present discounted value of downstream benefits.

To see specifically how this works, consider the vintage capital model from the previous section with $\gamma\rho > \beta$. Recall, that the growth of consumption was given by $g_c = (\delta\rho\gamma)^{1/(1+\theta)}$. The first order condition is that the consumer must be indifferent between consuming in period t and $t+1$. Consequently, the relative price of the consumption commodity between two periods must be $\gamma\rho$, and the equilibrium present value price p^*_{zt} of the consumption commodity is

$$p^*_{zt} \propto (\gamma\rho)^{-t}.$$

A unit of vintage t capital can produce γ^t units of the consumption commodity at time $t+1$. The entrepreneur who introduces this new kind of capital has a claim to its entire output. Competition between different entrepreneurs forces profit to zero, so the price of vintage t capital at time t, $p^*_{\kappa_{tt}}$, is

$$p^*_{\kappa_{tt}} = \frac{\rho^{-t-1}}{\gamma}.$$

An entrepreneur who attempted to reproduce his existing vintage of capital, would make a negative profit at these equilibrium prices. In this sense, the competitive pressure from other entrepreneurs forces each one to innovate in order to avoid a loss.

As in theories of monopolistic competition and other theories of innovation, new technologies are introduced because of the role of individual entrepreneurs in seeking out profitable opportunities. Unlike those theories, the entrepreneur does not actually end up with a profit. Because of competition, only the owners of factors that are in fixed supply can earn a rent in equilibrium. When a valuable innovation is introduced, it will use some factors that are in fixed supply in that period. Those factors are likely to earn rents. If you are good at writing operating systems code when the PC technology is introduced, you may earn some huge rents, indeed. In principle, this model allows a separation between the entrepreneurs who drive technological change by introducing new activities, and the owners of fixed factors who profit from their introduction. However, it is likely in practice that they are the same people.

Pricing of New Commodities

Recursive Arbitrage Pricing

We turn now to a broader examination of pricing. Recall our definition of supporting prices for consumption goods. We used their characteristics' content and the marginal utilities of those characteristics at an allocation $x^* \in \tilde{X}$. The price p_{nt} of good n at time t, when $x^*_{nt} > 0$ is

$$p^*_{nt} \geq \delta^{t-1} Du(Cx^*_t)C_n.$$

These supporting prices can be used to derive "no arbitrage prices" for new goods that are exclusively consumed: these goods are priced according to the characteristics they contain. Similarly, goods which produce characteristics and can also be used as inputs, can be priced according to the characteristics they contain, provided that it is optimal to consume these goods in equilibrium. Such pricing by arbitrage puts a natural bound on the equilibrium prices of new goods and, through it, on the size of the rents accruing to their owners. To summarize:

Theorem *Let λ^*, k^*, x^* be a competitive equilibrium supported by the price sequence p^*. Consider a period $t \in \{1, 2, \ldots\}$ and a good n which is viable at t. Let C_n be its characteristic vector and denote with p^*_{nt} the price of this good at time t. Assume there exists a collection of goods $\{n_1, \ldots, n_{n'}\}$, which are consumed in positive quantity at time t and have characteristic vectors C_{n_j} such that*

$$C_n = \sum_{j=1}^{n'} \alpha_j \bullet C_{n_j}, \qquad \alpha_j \in \Re, \ \ \forall j.$$

Then

$$p^*_{n,t} \geq \sum_{j=1}^{n'} \alpha_j \bullet p^*_{n_j,t}$$

with equality if good n is consumed in positive quantity at time t.

Proof: Follows directly from the supporting prices condition. QED

We are left with pure intermediate or capital goods. In the simple case in which the intermediate good is the sole input of an activity with an output that can be priced by arbitrage, the zero profit condition provides a straightforward pricing equation. In general, though, by means of the zero profit condition one, can only price bundle of inputs in terms of bundles of outputs, as more than one input is used in any given activity and we are also allowing for joint production. Then a set of simultaneous equations, one for each activity, must be solved to yield the prices of the intermediate goods. This is the general case, which needs to be considered in some detail.

Let us assume that in equilibrium all characteristics are consumed. In period t, consider the vector of M_t inputs $\kappa_t \in \Re^{M_t}_+$ which are pure intermediate goods: that is they can be used as inputs, but do not directly produce characteristics. Let $r_t \in \Re^{M_t}_+$ be their price vector, to be determined. Denote with $z_t \in \Re^L_t$ the vector of inputs which are also consumption goods. Let their prices be $q_t \in \Re^{L_t}_+$, which we take as given in the light of the previous discussion. Assume A_t linearly independent activities are operated. Partition the input vector of activity a as $k(a) = (\kappa(a), z(a))$.The zero profit condition requires that

$$r_t \bullet \kappa(a) = (r_{t+1}, q_{t+1}) \bullet y(a) - q_t \bullet z(a)$$

for $a = 1,\ldots,A_t$. Clearly, when $A_t \geq M_t$, the solution is a straightforward matrix inversion. In this case

$$r_t = B^1_t r_{t+1} + B^2_t q_{t+1} - B^3_t q_t, \qquad q_t = \delta^{t-1} Du(Cx^*_t)C.$$

where B^1_t, B^2_t, B^3_t can be calculated by matrix inversion. The former equation can be iterated forward. By making appropriate use of the transversality condition, we recover the traditional “net present value of future utilities flow” formula for pricing capital goods

$$r_t = \sum_{\tau=0}^{\infty} \left[\left(\Pi^{\tau-1}_{j=0} B^1_{t+j-1} \right) (B^2_{t-1} - B^1_{t+\tau-1} B^3_t) \right] q_{t+\tau}$$

where B^1_{t-1} is understood to be the identity matrix and B^2_{t-1} is zero.

When $A_t < M_t$ only A_t bundles of pure intermediate goods can be uniquely priced by the same method. This implies a certain “indeterminacy” of individual prices. Such indeterminacy concerns only prices of pure intermediate goods during the initial period. It is the outcome of the interaction between the input-output nature of linear activities and the completely inelastic supply of the pure intermediate goods in any given period.

Example: **Arbitrage Pricing**

Consider an economy in which there are two characteristics, so $J = 2$, three consumption commodities and one type of labor. The utility function over the two characteristics is a symmetric Cobb Douglas, i.e. $u(c_1, c_2) = (c_1)^{1/2}(c_2)^{1/2}$. The three consumption goods, z_1, z_2 and z_3, have the following vectors of characteristics:

$$C_1 = [1,0] \qquad C_2 = [0,1] \qquad C_3 = [\epsilon, 1]; \qquad \epsilon > 0$$

The commodity vector is therefore $x = [z_1, z_2, z_3, \ell]$. To economize on notation let χ^z_i denote the three-dimensional vector with one in the position of consumption good i and zero elsewhere. There are three potential activities, one for each consumption good. They are

$$a_{z_i} = [0, 1; \chi_i^z, 1] \qquad i = 1, 2, 3.$$

In words: labor can produce any of the three consumption goods, on a 1-1 basis while also reproducing itself.

Let this economy begin with an endowment of 2 units of labor. The set of initially available activities is $A_0 = \{a_{z_1}, a_{z_2}\}$. So, at the beginning, the third consumption good is not viable.

As long as $A_t = A_0$ the optimal production plan is

$$\lambda(a_{z_1}) = \lambda(a_{z_2}) = 1.$$

The supporting prices for the two consumption goods are

$$p_{z_1,t+1} = p_{z_2,t+1} = \frac{\delta^t}{2}, \qquad t = 0, 1, \ldots.$$

The zero profit condition can be applied to derive the equilibrium prices of labor

$$w_{t+1} = w_t - p_{z_i,t+1}, \quad i = 1, 2; \quad t = 0, 1, \ldots.$$

The transversality condition or, which is the same, the intertemporal budget constraint, yields

$$w_0 = \frac{\delta}{2(1-\delta)}, \quad \text{from which} \quad w_t = \frac{\delta^{t+1}}{2(1-\delta)}$$

Now consider what happens when the set of available activities is enlarged. Our interest here is not in the transition path and the oscillations in the value of aggregate output innovations may bring about (which, we note, is also interesting and left to the reader). We will look directly at the new steady state. Let $A_T = \{a_{z_1}, a_{z_2}, a_{z_3}\}$. There are still only two units of labor available, which implies that, in total, at most two units of the three consumption goods can be produced. In equilibrium, we will have $z_2 = 0$ as the third consumption good costs as much labor as the second but provides a strictly greater vector of characteristics. Hence, after the transition period, $\lambda(a_{z_2}) = 0$ and the two units of labor are allocated to the production of z_1 and z_3.

Maximization of steady state utility gives the optimal production plan. Along this, an amount equal to 1 of the first characteristic and an amount equal to $1/(1-\epsilon)$ of the second are produced and consumed in each period $t = T+1, T+2, \ldots$

$$\lambda(a_{z_1}) = \frac{1-2\epsilon}{1-\epsilon},$$

$$\lambda(a_{z_2}) = 0,$$

$$\lambda(a_{z_3}) = \frac{1}{1-\epsilon}.$$

The supporting prices for the three consumption goods, and the labor input can be computed once again by straightforward application of our decentralization theorem. Write $\eta = (1-\epsilon)^{1/2} < 1$. Then, for $t = T, T+1, \ldots$:

$$p_{z_1,t+1} = p_{z_3,t+1} = \frac{\delta^t}{2\eta},$$

$$w_{t+1} = w_t - p_{z_i,t+1}, \quad i = 1,2,3.$$

The price of the second consumption good is a little tricker, because it is not actually produced or consumed. From the first order condition for consumers, a lower bound on the price is

$$p_{z_2,t+1} \geq \frac{\delta^t \eta}{2},$$

otherwise consumers would demand to consume good 2. However, the price could be higher than this, and the activity producing the second good we still earn a negative profit, so we would still have an equilibrium. If we adopt the standard convention that for pure consumption goods, price is equal to the lowest equilibrium price, then the inequality becomes and equality, and in addition, the arbitrage pricing theorem holds. We will adopt this convention for the remainder of the paper.

From the latter and the intertemporal budget constraint, an explicit wage rate is obtained in each period

$$w_t = \frac{\delta^{t+1}}{2\eta(1-\delta)}.$$

At the new equilibrium prices the activity a_{z_2} makes negative profits

$$\pi_t^{z_2} = \frac{\delta^t \eta}{2} - \frac{\delta^t}{2\eta} < 0, \quad \text{as } \eta < 1;$$

which justifies the choice of $\lambda(a_{z_2}) = 0$.

Notice also that

$$p_{z_3,t+1} = \epsilon \bullet (p_{z_1,t+1}) + 1 \bullet (p_{z_2,t+1})$$

The price of the third consumption good is a linear combination of the supporting prices of the other two goods, with weights equal to the coordinates of its characteristics vector.

Profits Versus Rents

The previous example is a good starting point for discussing the way in which entrepreneurial innovations generate changes in the relative prices that may appear as rents to certain factors, how this affects income distribution across factors and in what sense these changes in relative prices should be considered as the “appropriate” competitive equilibrium rewards to entrepreneurial activity.

The assumption that there is only one type of labor, which is equally effective in producing any of the three kinds of consumption, implies that the social surplus generated by the introduction of the third consumption good is immediately appropriated by every member of society. The channel through which this productive surplus flows to the households, is the equilibrium price of labor which increases from $\delta^{t+1}/2(1-\delta)$ to $\delta^{t+1}/2\eta(1-\delta)$. The labor input employed in a_{z_3} is perfectly substitutable with the labor input employed in a_{z_1}, hence they must earn the same wage rate and the two capital goods must also be equally priced.

Nevertheless, the innovation is readily implemented as *everyone* has a private incentive to do so and constant returns to scale allow everyone to do so, arbitraging away profit opportunities.

Example: Profits, Rents and Income Inequality

We now examine what happens when we have differentiated labor. One type of labor may benefit from the introduction of a new technology, while the other does not.

Specifically, assume that there are two types of labor, $h = 1,2$, in equal amounts, $\ell^1 = 1$ and

$\ell^2 = 1$. The difference between ℓ^1 and ℓ^2 is that only the latter is able to produce z_3 Hence ℓ^2 can be used in any of the three activities $a_{z_1}, a_{z_2}, a_{z_3}$ with unchanged productivity, while usage of ℓ^1 is limited to the first two.

The competitive equilibrium when $A_t = \{a_{z_1}, a_{z_2}\}$ is the same as before: the two inputs are perfect substitutes, given the viable technology set, and earn the same income. When the set of viable activities expands to $A_T = \{a_{z_1}, a_{z_2}, a_{z_3}\}$, the equilibrium allocation now becomes

$$\tilde{\lambda}(a_{z_1}) = 1 - \frac{\epsilon}{2},$$

$$\tilde{\lambda}(a_{z_2}) = \frac{\epsilon}{2},$$

$$\tilde{\lambda}(a_{z_3}) = 1.$$

This is substantially different from the one we obtained before when the innovation was implemented. This is because the limited supply of ℓ^2 constrains the level at which the third activity can be operated. This implies it is now efficient to operate also the second activity. The different production plan is reflected in the equilibrium consumption of the two characteristics, which is now $\tilde{c}_i = 1 + \frac{\epsilon}{2}$ for $i = 1, 2$. Equilibrium prices now yield strictly zero profits for all three activities:

$$\tilde{p}_{z_1,t+1} = \tilde{p}_{z_2,t+1} = \frac{\delta^t}{2},$$

$$\tilde{p}_{z_3,t+1} = \frac{\delta^t(1+\epsilon)}{2}$$

for all $t = T, T+1, \ldots$. Our theorem on the pricing of new goods by arbitrage holds, and

$$\tilde{p}_{z_3,t+1} = \epsilon \bullet (\tilde{p}_{z_1,t+1}) + 1 \bullet \tilde{p}_{z_2,t+1}.$$

The combination of innovation and lack of substitutability between ℓ^1 and ℓ^2 drastically alters the distribution of income among factors of production. The wage of the first type of labor, ℓ^1 remains at its pre-innovation level

$$\tilde{w}_t^1 = \frac{\delta^{t+1}}{2(1-\delta)}$$

while that of ℓ^2 increases to

$$\tilde{w}_t^2 = \frac{\delta^{t+1}(1+\epsilon)}{2(1-\delta)}.$$

With heterogeneous labor, technological progress alters income distribution and, in this example, increases income inequality. Contrary to the previous case in which all inputs were perfect substitutes, the introduction of the new good generates a rent going to the only input which can produce the new commodity. The effect of this change in relative prices is large, as it transfers to ℓ^2 the total increase in aggregate output $\epsilon/2(1-\delta)$. But, this additional income accruing to ℓ^2 is not too large, at least if we look at it from the point of view of the incentives to innovate. Once the new activities are discovered the difference between total output with ℓ^2 and without ℓ^2 is exactly equal to the incrrease in ℓ^2's income. This corresponds to full private appropriation of ℓ^2 social contribution and generates the correct incentives for implementing the innovation. Innovating is therefore fully consistent with perfect competition and entrepreneurial rents are explained by changes in the relative prices of scarce resources.

A few extra remarks on the implications of this example may be in order. It should be obvious how to generalize it to the case in which there are many heterogeneous agents, each one endowed with a vector of different skills. Index these skills with $s = 1, \ldots, S$, and assume technological progress generates a sequence of consumption goods z_i, with $i = 1, 2, \ldots$, such that technology requires skills of indices $s \geq i$ to operate the activity a_{z_i}. Then one has a model of increasing specialization and division of labor in which the distribution of income changes over time because of the endogenous flow of economic innovations.

This type of framework is useful to formalize the idea that trade is more beneficial, and its volume increases, as individuals are made more heterogenous by technological progress. Assume there are heterogeneous individuals endowed with the different factors. Due to the assumption of constant returns to scale, agents are equally well off either altogether or in complete isolation when only the most primitive technology is available, as anybody can operate all activities and produce all viable goods. This is not possible at a more advanced stage when certain goods become available only by trading with other agents that either have the unique skill required to operate the new machine or can operate the same machines we can operate but at a lower unit cost. In this sense, the example shows that, in the absence of fixed costs, the division of labor *and* the size of the market are both limited by the degree of technological progress.

Pricing of Ideas

It is ordinarily thought that ideas or creative works are produced with a fixed cost, and that consequently, are inconsistent with perfect competition. There is a large literature on the appropriate type of monopoly (copyright or patent) governments should provide to permit the production of ideas and creations. It might seem then, that the competitive framework has little to contribute to the understanding of the production of ideas and creative works. Surprisingly, this is not the case: once we carefully model the element of time in production, we see that the issue is not one of fixed cost, but rather a sunk cost, and there is little reason to believe that competition is unable to deal with sunk costs. The issue, if there is one, revolves not around fixed cost, but rather around an indivisibility. As we shall see, even this indivisibility need not pose a problem for our competitive framework.

Our basic example is motivated by the production of music. The central idea is that the initial production of a song requires an investment of time over several periods. Following the initial production, that is, the composition of the song, the song may be inexpensively reproduced.

Example – Competition in Ideas

In this example there is a single characteristic so that $J = 1$. The period utility function is $u(c) = -(1/\theta)[c]^{-\theta}$ with $\theta > 0$. There are four commodities: raw labor, a single consumption good (music), and two different kinds of capital: intermediate capital (a half finished song) and final capital (a finished song). The consumption good may be converted into the desired characteristic on a 1–1 basis. We write a commodity vector $x = (\ell, z, \iota, \kappa)$ where the first entry is labor, the second consumption, the third intermediate and the fourth final capital. Prices are labelled by the respective commodity superscript. In the initial period the economy is endowed with one unit of labor and nothing else.

There are two ways of obtaining consumption, together they comprise a grand total of 6 activities. The first way is from labor directly: one unit of labor today generates one unit of consumption tomorrow. This may be thought of as performing an existing not terribly good song. The corresponding activity, activity 1, has the form $[1, 0, 0, 0; 0, 1, 0, 0]$.

The second way of obtaining consumption is more roundabout: it uses labor to obtain a half finished song (the intermediate capital) from which a finished song (final capital) is derived. The latter is an input both in reproducing itself and in producing the consumption good. We model the former by specifying activity 2 as $[1,0,0,0;0,0,1,0]$. Activity 3 uses intermediate capital to produce final capital, $[0,0,1,0;0,0,0,1]$. Activity 4 uses final capital to produce $\beta \geq 1$ units of final capital $[0,0,0,1;0,0,0,\beta]$. Activity 5 uses final capital to produce consumption $[0,0,0,1;0,\rho,0,0]$, where $\rho > 1$. Finally, activity 6 allows for storage of raw labor from one period to the next $[1,0,0,0;1,0,0,0]$.

The interpretation is that ρ represents the quality of the song, and β the (inverse of the) reproduction cost. The latter especially can be a large number: once the song is written production of additional copies may be relatively cheap.

For ρ and/or β large enough, the roundabout process dominates the direct one as a way of obtaining consumption from labor. Moreover, if there is any final capital in the economy the technology of producing final capital directly from itself dominates the roundabout method of production. In other words, a song will be written only once, using labor first and then its first draft (intermediate capital) to obtain a final version in period two. After that, additional copies of the half-written song (intermediate capital) are not useful: final capital reproduces itself at a rate $\beta \geq 1$, while consumption is obtained, at a rate $\rho > 1$, from that portion of final capital that is not reproducing itself.

By the same token, activities 1 and 6 are used in parallel only during the first two periods: to produce consumption for periods 1 and 2and to carry over labor from period zero to period one. From period three onward, consumption produced from final capital, via activity 5, becomes available. This implies that there are only two important types of production decision. First, which fraction $i \in [0,1]$ of initial labor to divert in the first period to the roundabout production of capital; and, second, what fraction ϕ of final capital to devote to the production of consumption once final capital becomes available. Naturally, we solve the second problem first.

Equilibrium quantities and prices

Because utility is CES the fraction $\phi \in [0,1]$ of final capital used to produce consumption does not depend on the current stock of final capital. As usual, we may solve

$$\beta(1-\phi) = (\delta\beta)^{1/(1+\theta)}.$$

where $\beta(1-\phi)$ is the long run growth rate of both consumption and final capital. The later is larger than one whenever $\beta > \delta^{-1}$, which we assume. For later use we compute

$$\phi = 1 - (\delta\beta^{-\theta})^{1/(1+\theta))} = 1 - \tilde{\delta}\tilde{\beta}^{-\theta}.$$

The restriction $\beta^{\theta} > \delta$, suffices to guarantee that $\phi \in (0,1)$. Altogether, we need $\beta^{\theta} > \delta > \beta^{-1}$, which rules out the case $\theta = -1$.

We now consider the tradeoff between labor used to produce consumption directly and indirectly. Notice that $1-i$ is the fraction of labor used to produce consumption directly in the first two periods. Equalizing the marginal utility of consumption in the first two periods requires $c_2 = \tilde{\delta}c_1$. Consequently, $i = 1-(1+\tilde{\delta})c_1 = 1-\hat{\delta}c_1$. Note that c_1 units of labor invested in activity 1 in period zero, yield, next period, an equal amount of consumption, with marginal utility equal to

$$\delta(c_1)^{-\theta-1}.$$

The fraction $i = (1-\hat{\delta}c_1)$ yields, in period 3 consumption with marginal utility equal to

$$\delta^3\rho\left[\rho\phi\left(1-\hat{\delta}c_1\right)\right]^{-\theta-1}.$$

Write $\tilde{\rho} = \rho^{\frac{1}{1+\theta}}$. We have

$$\frac{i}{1-i} = \frac{\tilde{\delta}^2 \tilde{\rho}^{-\theta}}{\phi \hat{\delta}}.$$

After substitution we get

$$i = \frac{\tilde{\delta}^2 \tilde{\rho}^{-\theta}}{\hat{\delta}(1 - \tilde{\delta}\tilde{\beta}^{-\theta}) + \tilde{\delta}^2 \tilde{\rho}^{-\theta}} \; .$$

Again, the restriction $\beta^\theta > \delta$ suffices to guarantee that the fraction i is in $(0,1)$.

Next, we compute the supporting prices. In every period, the price of consumption is proportional to marginal utility. In the first two periods

$$p_1^z = p_2^z \propto \delta \left[\frac{\hat{\delta}(1 - \tilde{\delta}\tilde{\beta}^{-\theta}) + \tilde{\delta}^2 \tilde{\rho}^{-\theta}}{1 - \tilde{\delta}\tilde{\beta}^{-\theta}} \right]^{1+\theta}.$$

For the early periods, the zero profit conditions imply

$$p_0^\ell = p_1^z = p_1^\ell = p_2^z; \quad p_2^\kappa = p_1^\iota = p_0^\ell; \quad p_3^z = \frac{p_2^\kappa}{\rho} = \frac{p_2^z}{\rho}.$$

For the other periods $t \geq 2$, zero profits imply

$$p_t^\kappa = \beta p_{t+1}^\kappa; \quad p_t^\kappa = \rho p_{t+1}^z$$

and, therefore,

$$\frac{p_{t+1}^z}{p_{t+1}^\kappa} = \frac{\beta}{\rho}.$$

Both the present value price of capital and consumption decreases at a rate $1/\beta$ per period, with the relative price determined by the ratio β/ρ. Further, $p_t^\ell = 0$ for all $t \geq 2$, as it is not needed anymore.

The usual condition $\beta^\theta > \delta$ is enough to guarantee that both the transversality condition

$$\lim_{t \mapsto \infty} p_t^\kappa \kappa_t = 0,$$

and boundedness of total utility along the optimal path

$$\sum_{t=1}^{\infty} \delta^{t-1} u(c_t) < \infty$$

obtain.

Let us now discuss the, implications of the model. To do this, it is useful to distinguish the case $\theta \in (-1, 0]$ from the case $\theta > 0$.

Elastic demand

This ocurs when $\theta \in (-1, 0]$. Notice first that the condition $\beta^\theta > \delta$ is more restrictive the closer θ is to -1. This makes sense: a high growth rate of consumption and capital together with high elasticity of intertemporal substitution in consumption lead to unbounded utility.

We now study the impact of an increase in the value of either ρ or β on the competitive equilibrium. We are especially interested in the impact of increasing β. This corresponds to lowering the reproduction cost, as would be the case, for example with modern digital technology for distributing music over the internet.

Consider first the case $\beta \to \delta^{1/\theta}$. The share of labor going into the production of intermediate capital is

$$i = \frac{\tilde{\delta}^2 \tilde{\rho}^{-\theta}}{\hat{\delta}(1 - \tilde{\delta}\tilde{\beta}^{-\theta}) + \tilde{\delta}^2 \tilde{\rho}^{-\theta}}$$

which converges to one, while the price of that labor (and the corresponding consumption) in period one (and two) converges to infinity. This case is especially significant, because it defies conventional wisdom: as the cost of reproduction declines, the competitive rents increase, despite the fact that many more copies are distributed. Yet the basic assumption is simply that it take some (small) amount of time to redistribute copies, and that demand is elastic. Notice that music producers and others have argued that with the advent of a technology for cheap reproduction their profits are threatened and increased legal monopoly powers are required. Yet this model shows that quite the opposite is possible: decreasing the reproduction cost makes it easier, not harder, for a competitive industry to recover production costs. Notice also that competition (unlike monopoly) does not require downstream licensing provisions: if each purchaser of music is permitted to freely reproduce and sell it makes no difference to the competitive equilibrium. The only “copyright” protection needed in this competitive industry is the right of first sale. The value of all subsequent sales is simply capitalized into the price of the first sale.

Similar comparative statics hold in the case where $\rho \to \infty$, in which case, again i goes to one and the initial price of labor goes to infinity

Inelastic demand

This case ocurrs when $\theta \in (0, \infty)$. We will examine it briefly, because it is of less practical relevance and because, in light of the previous discussion, most results should be obvious.

We now find that as $\rho \to \infty$, $i \to 0$. However, as $\beta \to \infty$ we have i approaching a finite limit

$$\lim_{\beta \to \infty} i = \frac{\tilde{\delta}^2 \tilde{\rho}}{\tilde{\delta}^2 \tilde{\rho} + \hat{\delta}}.$$

Even with inelastic demand, as the cost of reproduction falls, rent remains bounded away from zero.

Indivisibility

Our basic observation is that the fact that production is roundabout does not imply that there is a fixed cost. The initial capital must be produced; once it has been produced, production of the consumption good is relatively inexpensive. This means a sunk cost, as the cost of producing the initial capital is sunk at the time the consumption good is produced. But it is not a fixed cost, in the sense that we can maintain the assumption of constant returns to scale. Moreover, the fact that a good is an input into its own production, as is the case with the reproduction of music (or other creations or ideas) has no particular implication for competition.

Our constant returns to scale assumption does, however, have one implication, which may be thought of as less than realistic. We assume that if half as much labor is initially invested in the production of the initial song, it is the same as having half as many songs throughout the lifetime of the song. It may reasonably be argued that a song produced at half the effort is much less than half as many songs. This however, and despite appearances, is not an example of increasing returns to scale, but simply an indivisibility. That is, if less than a minimum initial amount of labor $\bar{\iota}$ is invested, then no song is produced at all. (If desired, an upper bound on the amount of labor can be added as well, but this does not imply an indivisibility, and can easily be modelled by adding a fixed factor.) Does this not invalidate our analysis, perhaps necesitating government grants of monopoly to operate this market?

A moment of reflection will show that our model is still quite relevant. The mere fact that we introduce an integer constraint into the model does not imply that it binds. Provided that

$$i = \frac{\tilde{\delta}^2 \tilde{\rho}^{-\theta}}{\hat{\delta}(1 - \tilde{\delta}\tilde{\beta}^{-\theta}) + \tilde{\delta}^2 \tilde{\rho}^{-\theta}} \geq \bar{\iota}$$

our analysis remains unchanged. Moreover, in the case of most relevance to modern policy discussions – elastic demand and large β – we observed that i is likely to be quite large. This means that the advent of cheap modern digital copying makes integer constraints on production of ideas and creations less likely to bind, and so weakens the case for government enforced monopoly.

Patterns of Innovation

The simple vintage capital model exhibits two basic sources of growth: capital accumulation and the introduction of new technologies. A basic feature of this model and our framework more broadly is efficiency: the competitive equilibrium maximizes the welfare of the representative individual. Can such a class of models capture growth anomalies: an initially poorer country in the long-run using a superior technology, or a modest increase in savings leading to a dramatic increase in growth? Certainly, models with fixed costs or other increasing returns can. A country that is far along the learning curve for an inferior technology may not wish to pay the fixed cost of introducing a completely different technology. A modest increase in savings can lower the marginal cost of investment leading to even greater investment. However, it is by no means the case that increasing returns are necessary for such conclusions, and insofar as there is empirical support for conclusions of this type, it cannot be taken as evidence of increasing returns to scale.

Our basic observation is that joint production can link the level of current consumption to the *kind* as well as level of future capital stock. As a result, joint production can play much the same role in our theory that fixed costs do under monopolistic competition: the need to produce a second commodity in order to use a particular technology acts as a kind of fixed cost, although it is consistent with constant returns, competitive decentralization and efficiency.

Savings and Innovation

We begin by examining the theoretical relationship between savings and growth. The basic result is that pattern of technological adoption in competitive equilibrium maximizes the long-term growth rate for each savings rate. An important corollary is that in the absence of joint production the technologies that lead to the most rapid rate of growth must be adopted. This is a kind of generalized convergence result: even in the presence of endogenous technological innovation all countries with the same technological possibilities wind up using the same technologies in the same sequence. After establishing these basic theoretical results, we illustrate through example how joint production can lead to growth anomalies through their counterintuitive consequences for savings rates.

Theorem *If a solution to the social planner problem λ^*, k^*, x^* is decentralized by prices p^* and a socially feasible plan λ, k, x satisfies*

$$\frac{p_t^* k_t}{p_t^*(k_t + x_t)} \leq \frac{p_t^* k_t^*}{p_t^*(k_t^* + x_t^*)}$$

then

$$\sum_{a \in A_t} p_{t+1}^* \lambda_t(a) y(a) \leq \sum_{a \in A_t} p_{t+1}^* \lambda_t^*(a) y(a)$$

Proof: From the initial condition $p_0^* k_0 = p_0^* k_0^*$. From the hypothesis on savings and the zero profit condition $p_{t+1}^* y(a) - p_t^* k(a)$, it follows recursively that $p_t^* k_t = p_t^* k_t^*$, that is, a plan that devotes no greater fraction of the value of output to savings never has a more valuable capital stock. From the zero profit condition we also have $\sum_{a \in A} \lambda_t(a)[p^*_{t+1}y(a) - p^*_t k(a)] \leq 0$ and $\sum_{a \in A} \lambda^*_t(a)[p^*_{t+1}y(a) - p^*_t k(a)] = 0$, from which the conclusion now follows directly.

QED

This theorem shows that higher total output than that from the socially optimum plan is possible only by a higher savings and investment rate. However, the measure of output is GNP plus the market value of the stock of capital after depreciation. Since the measurement of GNP requires some arbitrary conventions about what constitutes "new" capital, we cannot state an equivalent result for GNP. Moroever, an equivalent theorem cannot hold for consumption, it would always be possible to consume a great deal in a single period by diverting production out of the capital sectors. The next theorem shows, however, that the only way to increase consumption over the social optimum is to overinvest, or periodically divert production into consumption. In particular, if the investment rate is fixed, then no plan can have a higher long-run rate of consumption growth than the competitive equilibrium plan.

Theorem *If a solution to the social planner problem λ^*, k^*, x^* is decentralized by prices p^* and a socially feasible plan λ, k, x satisfies*

$$\frac{p^*_t k_t}{p^*_t(k_t + x_t)} \leq \frac{p^*_t k^*_t}{p^*_t(k^*_t + x^*_t)}$$

then, for all τ:

$$\sum_{\tau=T}^{\infty} p^*_\tau x_\tau \leq \sum_{\tau=T}^{\infty} p^*_\tau x^*_\tau$$

Proof: As in the previous theorem, $p^*_t k_t \leq p^*_t k^*_t$ at all t. For any feasible plan, the zero profit condition implies $p^*_{t+1}x_{t+1} + p^*_{t+1}k_{t+1} \leq p^*_t k_t$, with equality for an optimal plan. Together with the tranversality condition, this implies that $\sum_{\tau=T}^{\infty} p^*_\tau x_\tau \leq p^*_T k_T$, with equality for the optimal plan. This now yields the desired conclusion. QED

The previous two theorems show that the competitive equilibrium maximizes the growth rate for a given savings rate. What consequences does competitive equilibrium have for the rate at which new goods and technologies are introduced? To see how quickly new technologies are introduced, we consider using the same technologies as used in the competitive equilibrium, but diverting all output into investment. If this is possible, then our next theorem shows that this yields the highest possible level of output at each moment of time. It follows that no alternative method of introducing new technologies can yield a higher growth rate, and consequently that the competitive equilibrium introduces new technologies as quickly as is "desirable."

Theorem *If a solution to the social planner problem λ^*, k^*, x^* is decentralized by prices p^* and if there exists a socially feasible plan $\hat{\lambda}$ such that $\lambda^*_t(a) = 0$ implies $\hat{\lambda}_t(a) = 0$ and satisfies*

$$p^*_t \sum_{a \in \mathcal{A}} \hat{\lambda}_t(a)k(a) = p^*_t \sum_{a \in \mathcal{A}} \hat{\lambda}_{t-1}(a)y(a)$$

then for any socially feasible plan λ, k, x

$$\sum_{a \in \mathcal{A}} p^*_{t+1}\lambda_t(a)y(a) \leq \sum_{a \in \mathcal{A}} p^*_{t+1}\hat{\lambda}_t(a)y(a)$$

Proof: Consider any feasible plan λ, k, x and a solution to the social planner problem λ^*, k^*, x^*. Then $\sum_{a \in \mathcal{A}} \lambda_t(a)[p^*_{t+1}y(a) - p^*_t k(a)] \leq 0$ and $\sum_{a \in \mathcal{A}} \hat{\lambda}_t(a)[p^*_{t+1}y(a) - p^*_t k(a)] = 0$ follows from the assumption that $\hat{\lambda}$ diverts all output to investment. The desired inequality $\sum_{a \in \mathcal{A}} p^*_{t+1}\lambda_t(a)y(a) \leq \sum_{a \in \mathcal{A}} p^*_{t+1}\hat{\lambda}_t(a)y(a)$ follows.

QED

Like the previous two theorems, this theorem seems to point in the direction of convergence. However, it is as significant for what it does not say as for what it does say. It requires that it be possible to divert all output to investment using only the activities that are actually used in the competitive equilibrium. A moment's reflection will show that with joint production this may not be possible. To switch output into investment, it is necessary to replace activities that produce both capital and consumption goods with activities that produce only capital. If the activities that produce only capital were not used in the original equilibrium, the conclusion of the theorem can fail. We now explore by example the way in which this type of joint production can play a role in competitive theory very like the role that fixed costs do under monopolistic competition.

Joint Production and Innovation

Example–Comparative Advantage of Backwardness

We first construct an example in which we compare two countries facing the same technological possibilities, but with different starting conditions. Our goal is to give conditions under which the more advanced country, by being locked into an existing technology, actually grows more slowly in the long-run than the less advanced country. Notice that if the technology allows the diversion of all output into investment using the competitive activities, such an example is impossible.

There is one characteristic, preferences are CES, and, as in the simple vintage capital model, there are no fixed factors. There are three kinds of commodities: the consumption good, which provides the characteristic on a one-to-one basis, and two kinds of capital goods, each of which comes in many different qualities. Denote the infinite vectors containing the quality ladders for these two kinds of capital goods as κ^1 and κ^2 and write a commodity vector as $x = [z, \kappa^1, \kappa^2]$. We still use the indicator function χ_i to represent a unit of quality i capital stock; the position of χ_i in the commodity vector will tell if it is of type one or type two. So for, example, $[0, 0, \chi_3]$ is a vector with zero consumption, zero amount of κ^1 and one unit of κ^2 of the 3rd quality.

The set of activities is composed by the following triplets, for $i = 0, 1, \ldots$

$$a_{i1} = [0, \chi_i, 0; \gamma^i, \chi_{i+1}, 0], \quad \gamma > 1;$$

$$a_{i2} = [0, 0, \chi_i; \mu^i, 0, \chi_{i+1}] \quad \mu > 1;$$

$$a_{i12} = [0, \chi_i, 0; 0, 0, \beta\chi_1] \quad \beta > 0.$$

The first two activities represent the production possibilities of the first and second quality ladders; here each kind of capital reproduces itself and produces either γ^i or μ^i units of the consumption commodity at the same time. We assume that $\mu > \gamma$ so that κ^2 is a better kind of capital stock than κ^1: as it moves from one quality to the next the rate at which its ability to produce consumption increases is higher than that of κ^1. Hence, in the long run, the growth rate of consumption generated by the second quality of capital will always dominate the one generated by the first quality. There is also a technology that allows the conversion of the poor capital κ^1 into the better capital κ^2. Regardless of what quality of type 1 capital is available, a unit of κ^1 always converts into β units of κ^2, of quality $i = 1$.

We assume that the endowment at time $t = 0$ is initially 1 unit of quality i capital of the first type. The only decision is what fraction $\phi \in [0, 1]$ of this capital to convert to κ^2 by means of a_{i12}. Total utility, for given ϕ, is given by

$$- (1/\theta) \sum_{t=1}^{\infty} \delta^{t-1} [(1-\phi)\gamma^{t+i} + \phi\beta\mu^t]^{-\theta}.$$

The derivative of this expression with respect to ϕ is

$$\sum_{t=1}^{\infty} \delta^{t-1} [(1-\phi)\gamma^{t+i} + \phi\beta\mu^t]^{-\theta-1} [\beta\mu^t - \gamma^{t+i}],$$

and if any capital is to be converted, it must be that for $\phi = 0$ this expression is positive, or equivalently

$$\sum_{t=1}^{\infty} \delta^{t-1} [\gamma^t]^{-\theta-1} [\beta\mu^t - \gamma^{t+i}] > 0.$$

Letting $i \to \infty$ this expression approaches $-\infty$ so if the existing quality of capital of type 1 is sufficiently advanced, the superior kind of capital, type 2, will never be introduced. On the other hand, for β sufficiently large, this expression will be positive for $i = 0$, so that a less advanced starting point will result in a long run more advanced kind of capital stock and higher growth rate of consumption forever. An implication of this simple observation is that both catch-up and jumping ahead phenomena are easily modelled with constant returns and perfect competition.

This example also answers the question of whether initial conditions may matter for long-term growth when there is perfect competition. In this model, despite assuming that exactly the same technologies are feasible and that preferences are identical, the different initial conditions lead to diverging long-term growth rates. It is easy to tilt the example around showing that, under appropriate circumstances, poorer countries may be the loser and grow at a lower growth rate for extended periods of time or even forever. For the sake of brevity we will only sketch the intuition here. Assume that the switch-over activity satisfies

$$a_{i12} = \left[0, \frac{\chi_i}{\beta^{(i-M)}}, 0; 0, 0, \chi_{i-L} \right].$$

with $\beta > 1$ $M, L \geq 0$. That is to say: more advanced countries have lower transition costs and/or can switch to a more advanced quality of the new technology. Given a value for the quality index i in the initial period, one can select triples β, M, L to obtain any pattern of behavior: from immediate adoption, to fast adoption, slow adoption or no adoption for a number of periods. The latter technology can be modified further by introducing a labor saving mechanism and an activity that allows for endogenous growth of the labor force. Then a less advanced country, with a high enough growth rate of the labor force, will postpone forever the adoption of the more advanced type of capital. footnote

Finally, we should also point out that this example can be generalized to explain phenomena which, in the literature, are often attributed to the existence of externalities and market inefficiencies. What we have in mind is the so-called "path dependence" literature, (see Arthur [1989], David [1985] or Krugman [1991]). The latter posits that a number of historical episodes footnote can only be explained by appealing to external effects, increasing returns and, as a consequence, allocational inefficiencies. In Boldrin and Levine [1997] we study various versions of our abstract model to argue that, as a matter of theory, those episodes are consistent with constant returns, no externalities and allocational efficiency.

Example: Role of Investment in Technological Progress

A phenomenon related to the issue of whether it is “too expensive” to introduce a new technology, is the question of the impact of a small change in preferences on growth. For example, with increasing returns to scale, a small increase in patience can lead to a large increase in long-run consumption, as the small initial increase in savings lowers the marginal cost of further investment through economies of scale. However, the previous example shows that in the competitive environment, similar “ratcheting up” effects are possible when a small increase in the savings rate allows a jump to an entirely different technology. Once again, increasing returns to scale are seen to play no essential role in explaining “growth anomalies” such as the dependence of growth rates (and growth rates of productivity and TFP as well) on saving/investment rates.

Recall from the previous example, the condition

$$\sum_{t=1}^{\infty} \delta^{t-1}[\gamma^t]^{-\theta-1}[\beta\mu^t - \gamma^{t+i}] > 0.$$

determining whether the new higher growth technology will be introduced. Notice that for low values of t $\beta\mu^t < \gamma^{t+i}$ holds, while the opposite is true for large t, because $\mu > \gamma$. Suppose that the parameters $\delta^*, \gamma^*, \beta^*, \mu^*$ are such that the overall expression is equal to zero. Notice that the long run growth rate of the “old technology” economy is γ since capital simply produces its next generation plus an ever increasing amount of consumption γ^i. Similarly, the growth rate of the “new technology” economy is μ. Consequently, a small increase in the subjective discount factor δ from a number slightly smaller than δ^* to a number slightly larger than δ^* causes the long-run growth rate to discontinuously change from γ to μ.

Conclusion

This paper studies a model of perfect competition in which endogenous technological innovations and entrepreneurial activity make sense. It is based on the idea that innovative activity takes places because, under competitive pricing, entrepreneurs appropriate the social value of their innovations. Changes in the relative prices of inputs and outputs are the channels through which the increase in social welfare is funnelled into private benefits. Technological change is neither exogenous to individual choices, nor constant, nor dependent upon the existence of external effects, increasing returns or monopoly power. It is the product of tireless search for profitable opportunities on the part of a large number of agents. It depends upon initial conditions and relative prices. Its adoption changes relative prices, income distribution, relative factor productivities and growth rates. In this sense, we provide here a theory of Total Factor Productivity, as advocated in Prescott [1998].

In this paper our focus has been on the theoretical determinants of the introduction of new technologies. Key to our finding is the role of joint production, which plays a role in competitive theory similar to that of a fixed cost in the theories of monopolistic competition. When capital and consumption are jointly produced, as for example when the production of consumption goods leads to new human capital, then initial conditions matter in the long run. As a consequence of the importance of initial conditions, we can have poverty traps, catch-up and falling-behind types of phenomena.

An important extension of this work is to adapt our examples to show how growth can be cyclical and that a balanced growth rate arises only as a statistical average among different, oscillatory, growth rates. To us this suggests, among other things, that a theory of long-run and short-run oscillations in aggregate and individual factors productivity may be built within the general framework we have proposed here.

We also show how competitive equilibrium prices ideas and inventions, and more generally, goods for which prototypes are produced only once followed by reproduction at low and constant marginal costs. It does not appear that fixed costs do or should play any role in this analysis. If there is an issue, it is with the indivisibility of ideas, and we point out that despite this indivisibility, perfect competition may well be able to deliver the goods, that is, a steady supply of new inventions, creations and ideas.

Appendix

We provide here the proof of the decentralization theorem

Theorem *Suppose assumptions* 1, 2, 3, 4 *and* 5 *hold. Suppose that* λ^*, k^*, x^* *is a feasible allocation given* k_0 *and that* $\sum_{t=1}^{\infty} \delta^{t-1} u(Cx_t^*) < \infty$. *Then the following three conditions are equivalent*:

(1) λ^*, k^*, x^* *solve the planner's problem for initial condition* k_0.

(2) *There exist prices* p^* *satisfying the zero profit condition and such that* x^* *solves the consumer maximization problem given* p^* *with* $\sum_{t=1}^{\infty} p_t^* x_t^* < \infty$.

(3) *There exist prices* p^* *satisfying the zero profit condition such that the pair* p^* *and* x^* *satisfies the first order conditions and the pair* p^* *and* k^* *satisfies the transversality condition*.

Proof: First we observe that if the zero profit condition holds then the transversality condition is true if and only if $\sum_{t=1}^{\infty} p_t^* x_t^* < \infty$. Indeed from the zero profit condition

$$p_0^* k_0^* - p_{T+1}^* k_{T+1}^* = \sum_{t=0}^{T} (p_t^* k_t^* - p_{t+1}^* k_{t+1}^*) = \sum_{t=0}^{T} p_{t+1}^* x_{t+1}^*$$

(**3**)**implies**(**2**) Suppose that these first order conditions and the tranversality condition are satsified. Under Assumption 4 there is an $N_t(k_0)$ such that if $n > N_t(k_0)$ the commodity n is not viable. Let $N_t \geq N_t(k_0)$. Define the T truncated utility function by $U^T(c) = \sum_{t=1}^{T} \delta^{t-1} u(c_t)$. Consider the problem of maximizing of $U^T(c) + p_{T+1}^* k_{T+1}^*$ subject to

$$c_t = Cx_t, \quad \sum_{t=1}^{T} p_t^* x_t + p_{T+1}^* k_{T+1} \leq \sum_{t=1}^{T} p_t^* x_t^* + p_{T+1}^* k_{T+1}^*.$$

and $x_n = 0$ for $n > N_t$. The truncated first order conditions

$$p_{nt}^* \geq \delta^{t-1} Du(Cx_t^*) C_n \text{ with equality unless } x_{nt}^* = 0$$

are sufficient for a solution to this problem. Since this is true for arbitrarily large N_t, x^* is also a solution to the truncated problem where $x_n \in X$. Suppose that x^* does not solve the infinite problem. Then there is a budget feasible $\hat{x}$ that yields more utility. The budget feasibility of $\hat{x}$ implies that the pair $\{\hat{x}_0, \hat{x}_1, \ldots, \hat{x}_T\}$, $k_{T+1} = 0$ is budget feasible in the truncated consumer problem. Since x^* is the optimum in the truncated problem, this in turn means that

$$U^T(Cx^*) + p_{T+1}^* k_{T+1}^* \geq U^T(C\hat{x})$$

However, $U^T(Cx^*) \to U(Cx^*)$ and from the transversality condition $p_T^* k_T^* \to 0$. It follows that

$$U(Cx^*) \geq \lim_{T \mapsto \infty} U^T(C\hat{x}),$$

which is the desired contradiction.

(**2**) **implies** (**1**) This is a standard first welfare theorem proof.

(**1**) **implies** (**3**) Suppose that λ^*, k^*, x^* is a solution to the planner problem for the initial condition k_0. For $p^T \in (\times_{t=0}^{T} \Re_+^\infty)$, let $p^T(k_0)$ denote the vector of prices of viable commodities only. Observe that λ^*, k^*, x^* solves the problems of maximizing $U^T(Cx)$ subject to social feasibility and $k_{T+1} \geq k_{T+1}^*$. Since by Assumption 4 $A_t(k_0)$ is finite this is a finite dimensional problem over the viable commodity space. By standard finite dimensional arguments, we can find a price vector $p^T(k_0)$ over the viable commodities so that the first order conditions are satisfied for those commodities and the zero profit conditions are satisfied. Note that the zero profit conditions need only hold for viable activities, and such activities can only use and produce viable commodities in positive amounts, so the prices of non-viable commodities are irrelevant to the zero profit condition. For non-viable commodities, we simply define

$$p_{nt}^T = \delta^{t-1} Du(Cx_t^*) C_n.$$

Our proof will be complete if we can show that as $T \to \infty$, p^T has a limit point (in the product topology), and that this limit point satisfies the tranversality condition. Since the prices of non-viable commodities do not depend on T they obviously converge. The components of p_0^T corresponding to non-zero elements of k_0 are bounded above by some B_0 and below by zero. From the zero profit condition, it follows that for each t there is a number B_t such that the largest component of p_t^T corresponding to a commodity viable at time t is less than or equal to $\prod_{i=0}^{t} B_i$. This shows the existence of a limit point p^* in the product topology; by construction, this limit point satisfies the first order conditions; it remains to show it satisfies the tranversality condition. Recall that it is enough to check that $\sum_{t=1}^{\infty} p_t^* x_t^* < \infty$. From the first-order condition, this will be true if

$$\sum_{t=1}^{\infty} \delta^{t-1} Du(Cx_t^*) Cx_t^* < \infty.$$

Since u is concave and bounded below by $u(0)$ we have that $u(Cx_t^*) \geq u(0) + Du(Cx_t^*)Cx_t^*$, and so

$$\sum_{t=1}^{\infty} \delta^{t-1} Du(Cx_t^*) Cx_t^* \leq \sum_{t=1}^{\infty} \delta^{t-1} [u(Cx_t^*) - u(0)] < \infty$$

QED

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