Gresham’s Law for Politicians☆

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Abstract

We study a model of electoral competition in which two politicians with different office motivations set party platforms and both politicians and grassroots can provide electoral effort. While the underlying structure of the model is asymmetric, we show that both parties have an equal chance of winning the election. In equilibrium, however, only the greediest politician - the more office motivated - matters for policy polarization and welfare: a Gresham’s law for politicians. The greedier is the greediest politician, the greater is polarization and the lower is welfare. Less interest in politics means also greater polarization and lower welfare.

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1. Introduction

The outcomes of U.S. national elections are evenly balanced between parties. From the beginning of the roughly 200 years since it began until the current time the Democratic party has won presidential elections roughly half the time. Such a strong empirical regularity calls out for a strong explanation and the obvious explanation is that party platforms adjust so as to give both parties an equal chance. This is the message delivered in a simple standard Hotelling model applied to politics also known as the Downsian model: office oriented politicians locate themselves at the median voter position to maximize their chance of winning. Unfortunately this model makes the strong counterfactual prediction that there is no policy polarization - that is both parties choose the same centrist platform.

We introduce a simple model of two political parties competing in a single election. Each party has a politician, grassroots and voters. Politicians are office motivated, grassroots care about the platform and voters care about both the platform and polarization. The model is fundamentally asymmetric because the politicians differ in how strongly office motivated they are. They act as Stackelberg leaders moving first, choosing a platform, then choosing a level of electoral effort. Grassroots move second and also choose a level of electoral effort. Voters move last and as in a follow-the-leader type of model act passively turning out in response to the effort provided by the politician and grassroots.

In this model politicians have an incentive to choose polarizing platforms to induce effort from grassroots that substitutes for their own. In the resulting equilibrium both parties are equally likely to win, platforms are polarized, grassroots disipate rent and suffer the consequences, and politicians make no effort. Furthermore, the degree of polarization is determined by the greediest of the two politicians, that is the more office motivated - a kind of Gresham’s law for politicians. The greedier is the greediest candidate, the greater is polarization and the lower is welfare. Furthermore, less interest in politics, due, perhaps,

\footnote{Our model captures in the simplest way the fact that political parties are typically made up of several layers with leaders at the top, grassroots organizers and turnout brokers in the middle, and voters at the bottom.}
to relatively good economic times, also leads to greater polarization and lower welfare.

We emphasize that a unique element of our model is the idea that a subset of voters are not simply driven like sheep by politicians but engage in self-organization: the grassroots matter. Perhaps nowhere was this more apparent than in the 2016 US Presidential election where the Democratic party was blindsided by grass-roots organizations that got out the vote for Donald Trump in the absence of much Republican party effort. In the subsequent Congressional election this effort was matched, despite all gerrymandering, by an even more striking Democratic grass-roots movement. The history of politics is full of grassroots movements ranging from labor unions to social clubs and just as party leaders put effort into turning out the vote so do the grassroots. There are many existing models of platform competition and while all have a role for both politicians and voters, none have a role for grassroots. In light of the increasing importance of GOTV campaigns in political elections,\textsuperscript{4} our grassroots model provides an empirically grounded alternative to the valence competition models that follow Stokes (1963) critique to the Downsian model.\textsuperscript{5}

In models such as Herrera, Levine and Martinelli (2008) politicians have both office motivation and ideological motivation. To explain balanced elections such a model must assume that political parties are symmetric - a fact that in a sense is the one that needs to be explained. We assume instead that politicians are purely office motivated and place the ideological motivation with the grassroots. There is a great deal of evidence for this. The triangulation of Clinton and Blair was widely criticized as opportunistic. George H. W. Bush conveniently switched positions on abortion when it advanced his political career, Boris Johnson thought that Brexit was a terrible idea before he was for it, and

\textsuperscript{4}See Green and Gerber (2019) for voting mobilization and grassroots movements in US. Enos, Fowler and Vavrek (2013) provide evidence that GOTV, by increasing the differences between voters and nonvoters, may lead to an increase in political inequality.

of course Trump was a liberal New York Democrat before becoming a conservative Republican. A related model assuming purely office motivated politicians in the valence competition literature is Ashworth and Bueno De Mesquita (2009). Their model, however, is a symmetric one in which politicians have an incentive for more polarized platforms because it causes a volatile electorate to focus on issues rather than valence competition, and so increases rents to politicians. We focus instead on the substitutability between the campaign effort of candidates and grassroots to turnout voters in the context of a simple three-stage asymmetric contest that leads to simple and sharp results concerning equilibrium.

2. The Model

We study a three stage political contest with two parties $k \in \{L, R\}$. Each party has a politician, a representative grassroots member and a mass of potential voters. In the first stage politicians choose platforms $x_k \in \mathbb{R}^+$. Here $x_k = 0$ represents a centrist policy while $x_k > 0$ contributes to polarization which is measured by $V = x_k + x_{-k}$. These platforms are observed, and in the second stage politicians choose electoral effort $e_k$. These efforts are observed, then in the third stage the grassroots choose their own electoral effort $E_k$. The party that provides the greatest combined effort $e_k + E_k$ mobilizes more voters and hence wins the election, or if the efforts are equal, each has an equal chance of winning.$^6$

The politician of party $k$ receives a reward $B_k > 0$ for winning and nothing for losing. This office motivation represents how much rent the politician expects to get from the office, either from power or from money. We presume that the cost to society of this type of self-interested behavior by politicians exceeds the benefit to the politician: certainly this is the usual moral and economic view of rent-seeking by politicians.$^7$ Hence we view $B_k$ as a measure of how “greedy” the candidate is. Letting $p_k$ be the probability that party $k$ wins, the utility of

\footnote{We do not explicitly model voter mobilization: see Levine and Mattoozi (2020) for a formal model and a review of the literature.}

\footnote{We do not explicitly model these costs viewing them as some orders of magnitude less than the costs and benefits of economic policies.}
a politician is $u_k = p_k B_k - e_k$.

The grassroots of party $k$, by contrast, care only about the platforms. They receive a reward that depends on the platform: $x_k$ for winning and $-x_{-k}$ for losing. Overall utility is $v_k = A(p_k x_k - (1 - p_k) x_{-k}) - E_k$. Here $A > 0$ is a parameter indicating how strongly grassroots care about the policy issues.

The voters care both about the platform and about the social conflict induced by polarization and receive utility $A(p_k x_k - (1 - p_k) x_{-k}) - V$. They turn out to vote, however, only in response to the incentives in the form of effort offered by the politician and grassroots.8

With respect to welfare, we regard the politician as having measure zero. By the welfare of the grassroots we mean the average utility between the two parties, and similarly for the welfare of the voters. Since the gains to one party are cancelled by the loss to the other the grassroots welfare is the negative of the average effort while the welfare of the voters is $-V$.

Our solution concept is subgame perfect equilibrium.

3. The Equilibrium

Define a greediest politician $w$ to be such that $B_w \geq B_{-w}$, that is the one who gets the greatest rent from office-holding. Note that if there is a tie both politicians are by definition greediest. By a least polarized equilibrium we mean any equilibrium with the least polarization $V$ in the set of equilibria.

**Theorem 1.** In any equilibrium both parties have an equal chance of winning. Furthermore, neither politician $k$ provides any effort, each gets utility $B_k/2$, with probability one polarization is at least $V \geq B_w/(2A)$ and grassroots welfare is the expected value of $-AV/2$. There is an equilibrium in which $x_w = 0$ and $x_{-w} = B_w/(2A)$, so in a least polarized equilibrium polarization is exactly $V = B_w/(2A)$. Least polarized equilibria are in first period pure strategies and $x_w \leq B_{-w}/A$.

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8This is the standard assumption in follow-the-leader type of models such as Shachar and Nalebuff (1999).
The theorem contains a number of insights. First, the welfare of both grass-roots and voters is proportional to the negative of polarization. Hence we may unambiguously identify greater polarization with lower welfare. This gives particular meaning to least polarized equilibria as these are exactly the welfare maximizing equilibria, and indeed, the politicians are themselves indifferent as to the level of polarization.\(^9\)

Turning, then, to least polarized equilibria, only the office-motivation of the greediest politician matters for polarization and the greedier is the greediest politician, the higher is polarization. Furthermore, the more the grassroots care about issues, the less is polarization and the smaller is the positive effect of greediness on polarization.

Finally, turning from welfare to distribution, we observe \(x_w \leq B_{-w}/A\), that is lower values of \(B_{-w}\) restrict the possibility of more extreme policy positions of the greediest politician \(w\). Less extreme positions of the greediest politician are bad for the grassroots and voters of their own party and good for the grassroots and voters of the other party. That is to say, grassroots and voters in the party \(-w\) led by the less greedy politician are potentially better off the less greedy their own leader is.

We prove this theorem by backward induction, working from the end of the game.

3.1. Third Stage Game

We start with the game between the grassroots.

**Proposition 1.** Let

\[
Q = \frac{(e_{-k} - e_k)}{AV}.
\]

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\(^9\) In fact politicians may also be mildly averse to polarization. First, they are concerned with their legacy - highly polarizing politicians are less likely to be remembered fondly. In a similar vein less polarization means the opposition is less likely to engage in character assassination. Real assassination could be also an issue: both the Lincoln and McKinley assassinations appear to have been motivated in part by the highly polarized political atmosphere. The recent attack on the US Congress is another example of violence against politicians that arose from political polarization.
In equilibrium if \( AV < e_{-k} - e_k \) then \( p_k = 0 \), otherwise \( p_k = 1/2(1 + \text{sign}(e_k - e_{-k})Q^2) \). If \( e_{-k} = e_k \) then \( v_k = -Ax_{-k} \).

Proof. Notice that this is an all-pay auction with complete information and (possibly) head starts. If party \( k \) wins the election for certain, a grassroots \( k \) gets \( Ax_k - E_k \) and if party \( k \) loses for certain, grassroots \( k \) gets \(-Ax_{-k} - E_k\). Hence the benefit of winning over losing is \( AV \), and \( k \) will provide this much effort to get a certain win over a certain loss. It follows that the amount that \( k \) is willing to bid is \( AV + e_k \) since \( e_k \) units of effort are provided by the politician for free. If we assume without loss of generality that \( e_{-k} \geq e_k \), then this is a linear all-pay auction in which \(-k\) has a head-start advantage of \( e_{-k} - e_k \).

From standard results on complete information all-pay auctions (see for example Hillman and Riley (2006) and Baye, Kovenock and De Vries (1998)) there is a unique equilibrium, both grassroots adopt mixed strategies, and grassroots \( k \) (the disadvantaged) in equilibrium gets the utility of zero effort and losing for sure.

The structure of the mixed strategies are known: we give a heuristic derivation.\(^{10}\) The lowest bid of \(-k\) is \( e_{-k} \) and the most \( k \) is willing to bid is \( AV + e_k \). Hence if \( e_{-k} > AV + e_k \) it follows that \( k \) loses for certain. This gives the first result concerning \( p_k \). Otherwise the distribution of both grassroots bids must be uniform of equal height between \( e_{-k} \) and \( AV + e_k \) since the marginal gain to each grassroots of a higher bid is the same. The remaining weight of the disadvantaged grassroots is a spike of height \( F_k(e_k) \) at his lowest possible bid \( e_k \), and for the advantaged grassroots a spike of height \( F_{-k}(e_{-k}) \) at his lowest possible bid \( e_{-k} \). Since the uniforms are identical the height of these two spikes must be equal, that is \( F_k(e_k) = F_{-k}(e_{-k}) = Q \). From this we can work out the height of the spikes. The disadvantaged grassroots provides zero effort and loses for certain with probability \( Q \) and with the remaining probability \( 1 - Q \) bids uniformly incurring an expected cost of \( (AV + e_{-k} - e_k)/2 \). With probability \( Q \) this bid wins and with probability \( 1 - Q \) this bid wins half the time. Multiplying the overall probability of winning by the benefit of winning over losing and

\(^{10}\)See Levine and Mattozzi (2020) for details.
equating to the expected cost, we see that

\[
\frac{AV + e_{-k} - e_k}{2} = \left(Q + \frac{1 - Q}{2}\right)AV = \frac{Q + 1}{2}AV
\]

so that \( Q = (e_{-k} - e_k)/AV \) as stated.

It remains to work out \( p_k \). This is \( 1 - Q \) times the probability of winning when bidding uniformly which we just computed as \( (Q + 1)/2 \). Hence

\[
p_k = (1 - Q)(Q + (1 - Q)/2) = (1/2)(1 - Q)(1 + Q) = (1/2)(1 - Q^2).
\]

Utility of the grassroots is \( v_k = A(p_kx_k - (1 - p_k)x_{-k}) - E_k = AV - Ax_{-k} - E_k \). If \( e_{-k} = e_k \) the contest is symmetric so there is complete rent dissipation meaning here that \( E_k = AV \) so that \( v_k = -Ax_{-k} \).

3.2. Second Stage Game

For the second stage game we define an equilibrium as peaceful if \( e_k = e_{-k} = 0 \) with probability one. Otherwise we call the equilibrium contested. Let \( G_k, G_{-k} \) denote the equilibrium strategies of the politicians, \( u_k(G_k, G_{-k}) \) their utility and \( p(G_k, G_{-k}) \) the probability of winning.

**Proposition 2.** In the second stage game a peaceful equilibrium exists if and only if \( AV \geq \max B_k/2 \). Furthermore if \( AV > \max B_k \) it is the only equilibrium. If \( AV < \min \{ \min B_k, \max B_k/2 \} \) then there is no pure strategy equilibrium. In any equilibrium \( u_k(G_k, G_{-k}) \geq B_k - B_{-k} - AV \). If \( B_{-k} \leq B_k \) then in any contested equilibrium \( u_{-k}(G_{-k}, G_k) < B_{-k}/2 \).

Notice that if polarization is low the equilibrium must be in mixed strategies: the existence of such an equilibrium is not in question as it follows from the Glicksberg fixed point theorem. While little is known about the structure of such equilibria, what matters is that Proposition 2 establishes a key fact: any contested equilibrium must be less advantageous to the less greedy politician than a peaceful equilibrium. This is key since, as the first part of Proposition 2 establishes, by creating enough polarization in the first stage a peaceful equilibrium in the second stage can be guaranteed.
Proof. By Proposition 1 the objective function is given by

\[ u_k(e_k, e_{-k}) = B_k p(e_k, e_{-k}) - e_k = \frac{1}{2} B_k \left( 1 + \text{sign}(e_k - e_{-k}) \left( \frac{e_{-k} - e_k}{AV} \right)^2 \right) - e_k. \]

A peaceful equilibrium exists if and only if when \(-k\) provides no effort \(e_{-k} = 0\) it is optimal for \(k\) also to provide no effort. In this case the objective function is convex in \(e_k\) so the optimum is either to provide no effort and get \(B_k/2\) or provide \(AV\) units of effort and get \(B_k - AV\). It follows that there is a peaceful equilibrium \(e_k = e_{-k} = 0\) if and only if \(AV \geq \max B_j, j \in \{k, -k\}\).

Next, we consider the uniqueness of peaceful equilibrium. By differentiating the objective with respect to own action we find

\[ \frac{\partial u_k(e_k, e_{-k})}{\partial e_k} = \frac{B_k}{(AV)^2} |e_k - e_{-k}| - 1 \]

Observe that no politician will provide more effort than \(\max B_k\) so we may restrict attention to \(e_k \in [0, \max B_j]\). Hence

\[ \frac{\partial u_k(e_k, e_{-k})}{\partial e_k} \leq \frac{B_k \max B_j}{(AV)^2} - 1 \]

and also for any cdf \(G_{-k}\) with support in \([0, \max B_j]\) we have

\[ \frac{\partial \int u_k(e_k, e_{-k}) dG_{-k}}{\partial e_k} = \int \frac{\partial u_k(e_k, e_{-k})}{\partial e_k} dG_{-k} \leq \frac{B_k \max B_j}{(AV)^2} - 1 \]

Hence if \(AV > \max B_j\) we have

\[ \frac{\partial \int u_k(e_k, e_{-k}) dG(e_{-k})}{\partial e_k} < 0 \]

implying that zero effort is optimal, that is, there can be no equilibrium that is not peaceful.

Turning to the existence of contested pure strategy equilibria, we observe that if both politicians provide the same level of effort \(\partial u_k/\partial e_k = -1\) so this is an equilibrium only if it is peaceful. If both provide different levels of effort then
the one \( k \) providing higher effort is on the convex part of the utility function so must provide effort \( e_{-k} + AV \) and win for sure. This implies that \( -k \) loses for sure so it must be that \( e_{-k} = 0 \). If this is to be optimal for \( -k \) it must be that
\[
\frac{\partial u_{-k}(0, AV)}{\partial e_{-k}} = \frac{B_{-k}}{AV} - 1 \leq 0,
\]
that is \( AV \geq B_j \) for one of the \( j \in \{k, -k\} \).

We turn to utility. Since \( -k \) will never provide more than \( B_{-k} \) units of effort \( k \) can win for certain by providing an effort of \( B_{-k} + AV \) yielding a utility of \( B_k - B_{-k} - AV \).

Finally, suppose that \( B_k > B_{-k} \) and let \( c_k(e_k) = e_k / B_k \) denote the linear cost of exerting effort relative to the value of the prize \( B_k \). From optimality of \( G_k \) and symmetry we have
\[
p(G_k, G_{-k}) - c_k(G_k) \geq p(G_{-k}, G_{-k}) - c_k(G_{-k}) = 1/2 - c_k(G_{-k}).
\]
By subtraction we have
\[
p(G_k, G_{-k}) - 1/2 \geq c_k(G_k) - c_k(G_{-k}). \tag{3.1}
\]
Reversing the role of the two politicians we also have
\[
p(G_{-k}, G_k) - 1/2 \geq c_{-k}(G_{-k}) - c_{-k}(G_k)
\]
or since one politician’s chance of winning is the other’s chance of losing
\[
p(G_k, G_{-k}) - 1/2 \leq c_{-k}(G_k) - c_{-k}(G_{-k}).
\]
Together with 3.1 this gives
\[
c_{-k}(G_k) - c_{-k}(G_{-k}) \geq c_k(G_k) - c_k(G_{-k}) = (B_{-k} / B_k) (c_{-k}(G_k) - c_{-k}(G_{-k})).
\]
Since \( B_{-k} / B_k < 1 \) it follows that \( c_{-k}(G_k) - c_{-k}(G_{-k}) \geq 0 \) hence it must also be the case that \( c_k(G_k) - c_k(G_{-k}) \geq 0 \). From equation 3.1 then \( p(G_k, G_{-k}) \geq 1/2 \).
If \( p(G_k, G_{-k}) > 1/2 \) then certainly \( u_{-k}(G_{-k}, G_k) < B_{-k}/2 \). Suppose instead that \( p(G_k, G_{-k}) = 1/2 \). This implies that if one politician provides zero effort for certain both do so, so that the equilibrium would be peaceful. Hence both provide positive effort with positive probability so both get less than \( B_j/2 \), \( j \in \{k, -k\} \).

3.3. First Stage Game

We can now prove the main theorem. Recall that \( B_w \geq B_{-w} \).

**Theorem.** [1] In any equilibrium both parties have an equal chance of winning. Furthermore, neither politician \( k \) provides any effort, each gets utility \( B_k/2 \), with probability one polarization is at least \( V \geq B_w/(2A) \) and grassroots welfare is the expected value of \(-AV/2\). There is an equilibrium in which \( x_w = 0 \) and \( x_{-w} = B_w/(2A) \), so in a least polarized equilibrium polarization is exactly \( V = B_w/(2A) \). Least polarized equilibria are in first period pure strategies and \( x_w \leq B_{-w}/A \).

**Proof.** First, we rule out the possibility of a first stage strategy that results with positive probability in an equilibrium that is not peaceful on the path. Here we use the fact from Proposition 2 that if \( AV > \max B_j \) the second stage equilibrium must be peaceful. If \( B_w > B_{-w} \) then \(-w\) gets less than \( B_{-w}/2 \) in a contested equilibrium, but can guarantee this amount by choosing \( x_{-w} > B_w/A \) leading to a peaceful equilibrium. If \( B_w = B_{-w} = B \) then in a contested equilibrium one of the two, \( k \), must get less than \( B/2 \) so would be better off choosing \( x_k > B/A \) and getting \( B/2 \) in the second stage. Since all equilibria are peaceful on the path it follows that politicians provide no effort. Moreover, from Proposition 2 both parties have an equal chance of winning giving the politicians utility \( B_k/2 \) and with probability one \( V \geq B_w/(2A) \). Grassroots welfare is also given in that result.

Second, we show that the lower bound on polarization is achieved by showing that indeed \( x_w = 0 \) and \( x_{-w} = B_w/(2A) \) can be supported as an equilibrium. To do this, we assign a peaceful equilibrium to the second stage whenever one exists. We apply Proposition 2 to see that for any level of polarization equal to or greater than the equilibrium level there is a peaceful equilibrium in the
second stage. In particular on the equilibrium path politician \( k \) gets \( B_k/2 \). Since deviations by \( w \) only increase polarization no advantage is derived. Moreover, \( -w \) gets \( B_{-w}/2 \) and by Proposition 2 there is no equilibrium of the second stage game which gives greater utility than this. Hence this constitutes a subgame perfect equilibrium.

Third, polarization can be equal to \( B_w/(2A) \) with probability one if and only if the politicians employ pure strategies in the first stage.

Finally, \( w \) can choose a platform of \( x_w = 0 \) and by Proposition 2 receive at least \( B_w - B_{-w} - Ax_{-w} \) in the second stage. As this must be less than or equal to the equilibrium utility of \( B_w/2 \) we see that \( x_{-w} \geq B_w/(2A) - B_{-w}/A \). In a least polarized equilibrium \( x_{-w} = B_w/(2A) - x_w \). Substituting for \( x_{-w} \) in the inequality we have \( B_w/(2A) - x_w \geq B_w/(2A) - B_{-w}/A \) giving the stated upper bound on \( x_w \).

4. Conclusion

What narrative can we tell? Herrera, Levine and Martinelli (2008) cite evidence that polarization fell during the twentieth century up until about 1980 then started rising. We may think of \( A \) as a measure of a country’s concern about politics. In the US for example the earlier part of the century was fraught with depression, war, and then cold war. After 1980 the economy boomed, war and cold war were on the way out, and indeed the entire world started experiencing a wave of prosperity unprecedented in history. It makes sense then that in the face of reduced risk and increased prosperity, concern about politics (\( A \)) declined and consequently polarization rose. Through the lenses of our simple model, the recent increase in polarization could be related to political leaders striving to transfer the cost of electoral effort to the voters. The less concerned about political issues voters are, the greater polarization is needed to goad them into action. Hence we no longer see issues of war, peace and prosperity, but rather minor divisive issues take center stage. The more greedy politicians are, the more they will exploit polarization. Since the consequences on voters depend on the greediest politician, our model suggests that high variance in potential candidates can be pernicious as much as one bad apple can spoil the entire
barrel.
References


