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Three Essays on Herding and Group Reputation

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by

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To my wife Qin, my son Derek

and especially to my parents

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Abstract of the Dissertation

Three Essays on Herding and Group Reputation

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This dissertation, in broad terms, focuses on the theoretical analysis of the mechanisms of human interaction in the context of sequential decision-making under imperfect information.

The first chapter analyzes a sequential decision model with one-sided commitment in which decision makers are allowed to choose the time of acting (exercising a risky investment option A) or waiting. The existing literature assumes an exogenous ordering of decision makers, in which only one decision maker moves in each period in an exogenously given order. If information previously aggregated dominates their own private information, individuals ignore their own private information and follow their predecessors – herding occurs. Consequently, their decisions are uninformative to others, which prevents information aggregation. Therefore, initial realization of signals can have long-term consequences and herd behavior is often error prone. My main question of inquiry is: if we allow decision makers to choose the time of acting or waiting, will herd behavior be more or less error prone? I characterize herd behavior under endogenous ordering and show that with endogenous ordering, if the number of decision makers is large and

decision makers are patient enough, at any fixed time, nearly all decision makers wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information.

The second chapter investigates the two-sided commitment case, in which decision makers have the third choice, exercising a safe investment option B. Results obtained in this case are similar to those found in the one-sided commitment case. However, one striking result is that with endogenous ordering and two-sided commitment, even though waiting forever is a dominated strategy, if the number of decision makers is large and decision makers are patient enough, decision makers wait too long.

The third chapter explores what group reputation is and models its formation and evolution. I define a player's group reputation as the belief others have about the characteristics of the group he belongs to, which is based only on group signals. A player's individual reputation is derived from his group reputation by adding individual signals. A model of group reputation of civil servants is constructed to identify the strategic behavior of potential bribers and civil servants, the corresponding levels of corruption, and possible anti-corruption policies along with their effects.

CHAPTER 1

Robust Herding with Endogenous Ordering and One-Sided Commitment

1.1 Introduction

How do people make sequential decisions under imperfect information? One may learn from his own experiences or from other people's choices. For instance, individuals currently using a particular software package may also have the choice of upgrading to a new software package. They may have some knowledge about the new software package. But if the new software package is brand new and private information is limited, individuals may be inclined to wait for other people to discourse more information about the newly released software before they take any action. If the information previously aggregated dominates their own private information, individuals ignore their own private information and follow their predecessors – **herding** occurs.[1] Herding prevents the aggregation of information. Therefore, the initial realization of signals can have long-term consequences and herd behavior is often error prone. The decisions of the first few individuals' can have a disproportional effect.

[1]Çelen and Kariv (2004) attempt to make the distinction between herding and information cascades. They point out that in a herd, individuals choose the same action; but they may have acted differently if the realization of private signals had been different. In an information cascade, individuals ignore their own private information and follow their predecessors. Thus, information cascades in Çelen and Kariv (2004) are equivalent to herding in this paper.

Bikhchandani, Hirshleifer, and Welch (1992), hereafter BHW, and Banerjee (1992) investigate herd behavior under **exogenous ordering**, in which the decision ordering is exogenously given and only one individual moves in each period. The restaurant example in Banerjee (1992) may fit the exogenous ordering setting.[2] But in many other cases, **endogenous ordering** which allows individuals to choose the time of acting or waiting may be more appropriate. For instance, when individuals decide to buy a new car or computer, they have the option to buy immediately or to wait. With endogenous ordering, there exist strategic interactions among decision makers. Due to the free-rider problem, some decision makers may have incentives to delay their decisions and learn from other decision makers, while others make decisions immediately if they feel confident that their decisions will produce desirable results. Furthermore, more than one individual can act or wait during the same period and consequently their decisions can be clustered together. Thus, under the endogenous ordering setting, the insight will be completely different from that under the exogenous ordering setting. Our main question of inquiry is: if we allow decision makers to choose the time of acting or waiting, will herd behavior be more or less error prone?

Continuing with the software upgrading example, there is a new software package A available for upgrading. Individuals are currently using a software package B. It is known that with some prior probability A is better than B. Each individual also gets a private signal indicating whether A is better or not. Upgrading to A is an irreversible choice. Once they upgrade to A, they are committed to their decisions.[3] But there is no commitment to continuing using

[2] In the restaurant example in Banerjee (1992), there are two restaurants next to each other. Individuals arrive at the restaurants in sequence. Observing the choices made by people before them, they decide on either one of the two restaurants.

[3] There exists extremely high "disruption costs" involved in upgrading. In other words, we could see this upgrade as a perpetual American call option. Individuals are free to exercise the option at any time they want. But once they exercise the option, they cannot reverse their

B. If individuals have not upgraded, they continue to have the option of doing so.[4] Thus, the software upgrading example belongs to the setting of **one-sided commitment**.

In contrast, the restaurant example in Banerjee (1992) is a **two-sided commitment** decision problem. Individuals choose between two restaurants. Choosing either one of the two restaurants is irreversible. Once an individual chooses one restaurant, he cannot go to the other any more. For exogenous ordering, one-sided commitment is equivalent to two-sided commitment because once an individual chooses A or B at his turn, he is out of the game and cannot change his decision any more. But for endogenous ordering, individuals in a one-sided commitment decision problem have two choices: A or B. If they choose A, they cannot change. If they choose B, they still have the option of choosing A later. Individuals in a two-sided commitment decision problem have three choices: A, B or wait. If they choose A or B, they cannot change. If they choose to wait, they still have the option of choosing A or B later. In other words, waiting is equivalent to choosing B in a one-sided commitment decision problem with endogenous ordering.

In this paper we concentrate on the one-sided commitment case.[5] We analyze an endogenous ordering sequential decision model in which decision makers are allowed to choose the time of acting (upgrading to the new software package A) or waiting (continuing using the current software package B). To emphasize the information aspect, we focus on pure information externalities: each decision maker's payoff only depends on his own action and the state of nature. Our main results are summarized below.

decision.

[4]Throughout the paper, we use the software upgrading example to illustrate our model.

[5]Its companion (Zhang 2007b) investigates the two-sided commitment case.

1. With endogenous ordering, we show the existence of a symmetric equilibrium with the following monotonicity property: in each period there exists a critical type of individual who upgrades with probability less than one; all types of individuals with private signals indicating a higher value of A upgrade with probability one; all others wait.

2. In this particular equilibrium, there is a strategic phase, followed by a herding phase. In the **strategic phase**, depending on their own private signals, some individuals upgrade, while others wait. In the **herding phase**, all the remaining individuals either upgrade immediately or wait forever regardless of their own private signals. Compared with the exogenous ordering setting, disclosure of public information has a completely different impact on the strategic and herding behavior of individuals. In particular, if the game is in the upgrade herding phase, all the remaining individuals upgrade immediately and the game ends in one period. Further disclosure of public information will not have any effect.

3. With endogenous ordering, if the number of individuals is large and individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. In this case, if individuals can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information.

There are some papers which investigate the decision problem with endogenous ordering. For example, Chamley and Gale (1994) investigate a discrete time investment model which assumes the timing of decisions is endogenous, that is, individuals try to find the best place in the decision-making queue. In their model, there are only two types of individuals: those with investment options and those without. Those individuals without investment options are assumed to be passive. In contrast, in our model we allow for a finite or an infinite number of types of individuals. Given one's own signals, each individual decides whether to upgrade immediately or to wait and learn the true value of the new software package A by observing other individuals' actions.

The rest of the paper is organized as follows. Section 1.2 begins with an example of two types of individuals in an attempt to capture our main idea. Section 1.3 provides the setup of a general model and shows the existence of a symmetric equilibrium with the monotonicity property. Then we characterize herd behavior under exogenous ordering and endogenous ordering and discuss our main results. Several extensions and modifications of the general model are presented in section 1.4 before we offer our conclusion in Section 1.5.

1.2 An Example

We begin with an example of two types of individuals who choose to either upgrade to the new software package A or to continue using the current software package B. If an individual continues using the current software package B, he gets a reservation utility V^0, normalized to zero. The benefit from A, denoted by V, is the same for all individuals and is either $1/2$ or $-1/2$, with equal prior probability. Each individual privately observes a conditionally independent signal about the true value of V. Individual i's signal μ_i is either H or L as described in the following table, where $p > 1/2$. The common discount factor is δ. Although the discount factor does not play a role in the decision making under the exogenous ordering setting, it does under the endogenous ordering setting.

Before characterizing and comparing the equilibrium results of exogenous and

Table 1.1: Signal Probabilities

	$Pr(\mu_i = H\|V)$	$Pr(\mu_i = L\|V)$
$V = 1/2$	p	$1 - p$
$V = -1/2$	$1 - p$	p

endogenous ordering settings, we describe some benchmark cases for comparison. If there are no interactions among the individuals, each individual makes a **self-decision** using his own private signal and the prior probabilities. The probability for each individual making the correct choice is p, the precision of the private signal. If there is a **social planner** who can gather the private information from all individuals, then based on all private signals and the prior probabilities, we can imagine that the probability for the social planner of making the correct choice is increasing in the number of conditionally independent signals. Certainly, in the **complete information** case, the true value of the new software package A is known and everyone makes the correct choice. In the other extreme case, if individuals make **random decision**, based on only the prior probabilities, then only half of the individuals will make the correct choice.

1.2.1 Setting I: exogenous ordering

The ordering of individuals is an exogenous sequence and known to all. Individuals differ in their positions in the queue and only one individual moves in each period. Each individual observes the actions of those before him. When it is his turn to make a decision, he decides to upgrade or to reject A according to current public information and his own private signal. With N individuals, the game ends in N periods. Following the tie-breaking rule in BHW, we assume that an individual indifferent between upgrading and rejecting A chooses to upgrade or to reject A with equal probability.[6]

Similar to the specific model in BHW, the equilibrium decision rule is described as follows. In period 1, the first individual rejects A if his signal is L and upgrades to A if his signal is H as the signal precision $p > 1/2$. In period 2, the second individual can infer the first individual's signal from his predecessor's decision. Based on his own private signal and the inferred first individual's signal, the second individual makes the following decision: if the first individual rejects A, he rejects A if his signal is L and rejects or upgrades to A with equal probability 1/2 if his signal is H; if the first individual upgrades to A, he upgrades to A if his signal is H and rejects or upgrades to A with equal probability 1/2 if his signal is L. In period 3, we have one of the following three situations: (1) if both predecessors reject A, then the rejecting herding phase starts – the following individuals reject A regardless of their own signals; (2) if both predecessors upgrade to A, the upgrade herding phase starts – the following individuals upgrade to A regardless of their own signals; (3) if one predecessor rejects A while the other upgrades to A, the third individual and the forth individual are in the same situation as the first individual and the second individual respectively. The following individuals are in the similar situation until the game ends in period N.

1.2.2 Setting II: endogenous ordering

Individuals are allowed to choose the time of acting (upgrading to A) or waiting (continuing using B). In any period t, each individual decides to wait or to upgrade to A if he has not upgraded to A yet. If he waits, he gets reservation utility $V^0 = 0$ and has the option of upgrading to A later.

[6]The tie-breaking rule matters for the efficiency of the exogenous ordering setting. In section 1.4.1, we discuss the general tie-breaking rule.

The equilibrium decision rule has the following properties (See the Appendix):

(i) **Period 1:** For the symmetric equilibrium, there exists a $\delta^*(N,p)$, which is decreasing in N and increasing in p. In period 1, type L individuals will wait to see type H's action. Type H individuals will upgrade to A for sure if $\delta \leq \delta^*(N,p)$. Otherwise, type H individuals will upgrade to A with some probability $0 < p_{H,1} < 1$, where $p_{H,1}$ is decreasing in δ and N, and increasing in p.

(ii) **Large Number of Individuals:** If the number of individuals is large, there exists a $\delta^*(p) = \lim_{N\to\infty} \delta^*(N,p)$. If $\delta \leq \delta^*(p)$, type H individuals will upgrade to A for sure in period 1. If $\delta > \delta^*(p)$, at any fixed time, nearly all individuals wait due to the negligible information disclosed.

Intuitively, in period 1, for type L individuals, the expected benefit from upgrading to A is $(\frac{1}{2} - p) < 0$. The expected benefit from waiting is greater than or equal to the benefit from waiting forever, which equals zero. Therefore, a type L individual will wait for sure in period 1. For type H individuals, in period 1, the expected benefit from upgrading to A is $(p - \frac{1}{2}) > 0$. If no one else upgrades to A, the expected benefit from waiting is equal to the benefit from waiting forever, which equals zero. A type H individual will upgrade to A if no one else upgrades. For a symmetric equilibrium, this means $p_{H,1}$ (the probability of type H individuals upgrading to A in period 1) is greater than zero. If discount factor δ is low enough, type H individuals will upgrade to A for sure. As the number of individuals N increases, precision of signals p decreases, and discount factor δ increases, type H individuals have a higher incentive to wait and $p_{H,1}$ decreases.

For the case of large number of individuals, if $p_{H,1}$ is strictly greater than 0, by the Law of Large Numbers, the true value of the new software package A will be (approximately) revealed in the second period. In this case, if individuals are patient enough, then all individuals will wait in period 1 such that $p_{H,1}$ is equal to 0. This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game. The probability of type H individuals upgrading to A in period t, which is denoted by $p_{H,t}$, is equal to 0 or approximately equal to 0. Consequently, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed.

1.2.3 Expected Number of Correct Choices

Let $X(N)$ represent the expected number of correct choices with N individuals in the game. Subsequently, $X(N)/N$ is the average expected number of correct choices. We present the following results.

Result 1.1 *(See the Appendix) For the example above, given δ and p,*

> *(i)* ***Impatient Individuals:*** *If $\delta \leq \delta^*(p)$, the equilibrium with endogenous ordering is more efficient in terms of inducing a larger expected number of correct choices.*
>
> *(ii)* ***Patient Individuals:*** *If $\delta > \delta^*(p)$, there exists an N^*, such that if $N < N^*$, the equilibrium with endogenous ordering is more efficient in terms of inducing a larger expected number of correct choices; there also exists an N^{**}, such that if $N > N^{**}$, the equilibrium with exogenous ordering is more efficient in terms of inducing a larger expected number of correct choices.*

Figure 1.1 sketches out the implication of Result 1.1. We can see that if N is large and individuals are patient (stage III in the figure), endogenous ordering is worse than self-decision, not to mention exogenous ordering.

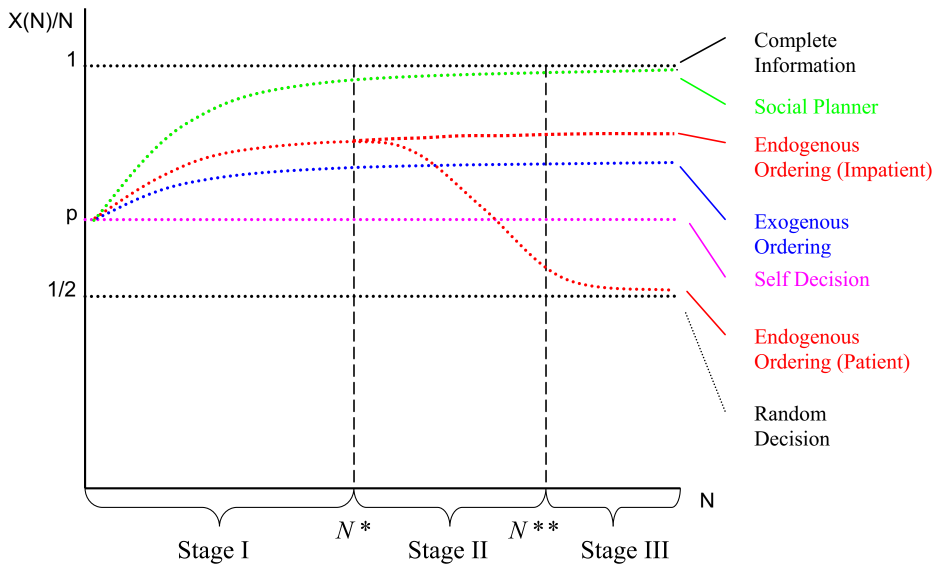

Figure 1.1: Average Expected Number of Correct Choices

1.3 The General Model

In this section, we first provide the basic setup of our general model. Then we characterize herd behavior under exogenous ordering and endogenous ordering.

1.3.1 Basic Setup

There are N individuals. All are rational and risk neutral. There is a new software package A available for upgrading. Individuals currently use software package B. Assume that the true value of A, denoted by V, is chosen by nature at the beginning of the game, and is unknown to the individuals.[7] Individuals only know V follows some prior distribution $F_0(V)$, with density $f_0(V)$. To emphasize the information aspect, we concentrate on pure information externalities: each individual's payoff only depends on his own action and the state of nature.

We focus on the case that upgrading to A is an irreversible binary choice.[8] The indivisibility of the action space is important. As in Banerjee (1992), since the choices made by individuals are not sufficient statistics for the information they have, the error prone herding can occur.[9]

At the beginning of the game, individual i in the market freely observes some conditionally independent private signal $\mu_i \in [\underline{\mu}, \overline{\mu}]$, which follows some distribution $F(\mu_i|V)$, with density $f(\mu_i|V)$. Assume individuals are more likely to get a higher private signal (indicating higher value of A) if the underlying V is higher.

Assumption 1: $F(\mu_i|V)$ satisfies the *Monotone Likelihood Ratio Property*

[7] Rosenberg (1976) points out that there exist two types of technological uncertainty. First, when an innovation is introduced, it may have some imperfections: "Innumerable 'bugs' may need to be worked out. The first user often takes considerable risk." In addition, current innovation could be improved further in the future. There are two possible situations for the future possible improvement: expected or unexpected. If it is expected, then it only increases the benefit from waiting by some constant amount. If it is unexpected, it will not affect the strategic interactions of the current game until it happens. Thus, we ignore the second type of technological uncertainty here. When we investigate the switch from one innovation to another, the future improvement, either expected or unexpected, could be incorporated.

[8] There exists extremely high "disruption costs" involved in upgrading. In other words, we could see this upgrade as a perpetual American call option as in Grenadier (1999). In Grenadier (1999), decisions are made in continuous time and there is a state variable, which follows some exogenous continuous time stochastic process. In this paper, we assume discrete time decision and no exogenous state variable.

[9] Banerjee (1992) assumes a continuous action space and gets similar herding results as BHW. This is due to the degenerate payoff function as pointed out by BHW. Park (2001) assumes perfect observability. Therefore, in his model players share the same information and hidden information is not an issue.

(MLRP)[10] with respect to V, i.e.

$$\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)} \quad \text{increasing in} \quad \mu_i \quad \forall V_1 > V_2$$

If individual i upgrades to A in period t, then in the following periods, everyone knows individual i upgrades to A in period t. The public information available at the beginning of period t is denoted by h_t, which includes the prior information of V, actions and the equilibrium strategy profile of all individuals before t. If an individual does not upgrade to A, he gets reservation utility V^0, normalized to zero. The common discount factor is δ.

1.3.2 Herd Behavior with Exogenous Ordering

If we assume the ordering of individuals is exogenous, in which only one individual moves in each period in an exogenously given order, then there are no strategic interactions among individuals. When it is one's turn to make a decision, he decides whether to upgrade or to reject A given the current public information and his own private signal.

The equilibrium decision rule is a sequence of critical values

$$\{\mu_t^*(h_t)\}_t$$

such that the individual making the decision in period t upgrades to A if his private signal $\mu_t > \mu_t^*(h_t)$; otherwise, he rejects A.[11] We can see this sequence of critical values is not monotone. If the individual in period t upgrades to A,

[10] Landsberger and Meilijson (1990) point out that this property holds for exponential type families (binomial with the same number of trials, normal with equal variances, etc.) as well as for some non-exponential cases such as uniform with the same left endpoint.

[11] For notation simplicity, we assume the following tie-breaking rule: if an individual i is indifferent between upgrading and rejecting A, he rejects it whenever $\mu_i \in (\underline{\mu}, \overline{\mu}]$ and upgrades whenever $\mu_i = \underline{\mu}$.

which indicates $\mu_t > \mu_t^*(h_t)$, this is "good" news for the individual in period $t+1$. Thus, in period $t+1$, $\mu_{t+1}^*(h_{t+1}) \leq \mu_t^*(h_t)$. Conversely, if the individual in period t rejects A, $\mu_{t+1}^*(h_{t+1}) \geq \mu_t^*(h_t)$.

The game is in the strategic phase when the sequence of critical values $\{\mu_t^*(h_t)\}_t$ fluctuates in between $\underline{\mu}$ and $\overline{\mu}$. In the strategic phase, each individual's decision depends on both the current public information and his own private signal.

Once the sequence of critical values $\{\mu_t^*(h_t)\}_t$ "breaks" either one of the boundaries, herding occurs. The upgrade herding phase starts in period τ if $\mu_\tau^*(h_\tau) = \underline{\mu}$. The individual in period τ will upgrade to A regardless of his own private signal. His decision is, therefore, uninformative to others. Thus, $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \underline{\mu}$ $\forall t > \tau$ (see figure 1.2). All the following individuals will upgrade to A. Similarly, the rejecting herding phase starts in period τ if $\mu_\tau^*(h_\tau) = \overline{\mu}$. All

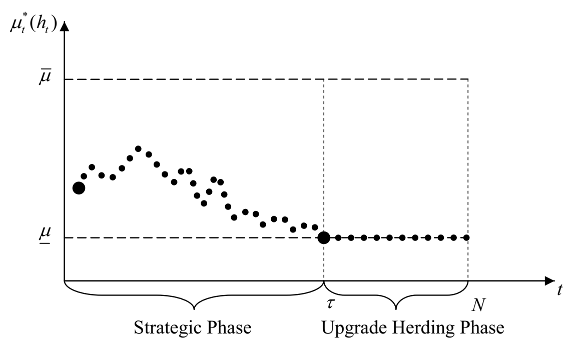

Figure 1.2: Upgrade herding with exogenous ordering

the following individuals will reject A and $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \overline{\mu}$ $\forall t > \tau$ (see figure 1.3).

Since public information disclosed only needs to offset the information from the last individual's action before the herding phase starts, both upgrade herding and rejecting herding are not robust to the public disclosure of information. If in

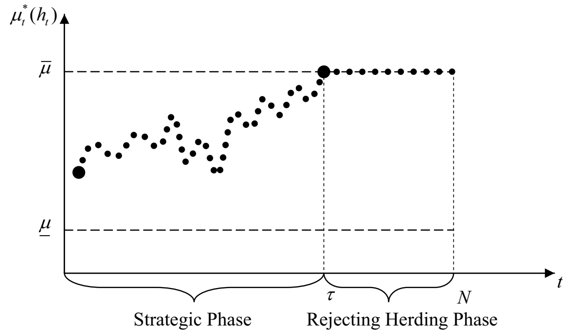

Figure 1.3: Rejecting herding with exogenous ordering

a certain period $N \geq t \geq \tau$ there is some public information disclosed such that $\underline{\mu} < \mu_t^*(h_t) < \overline{\mu}$, then the strategic phase starts again.

1.3.3 Herd Behavior with Endogenous Ordering

If we allow the individuals to choose the time of acting (upgrading to the new software package A) or waiting (continuing using the current software package B), there exist strategic interactions among the individuals.

The timing of endogenous ordering is as follows:

> In period 1, each individual decides whether or not to upgrade to A. If he does not upgrade to A in period 1, he gets reservation utility $V^0 = 0$ and has the option of upgrading later.
>
> In period 2, all the remaining individuals decide to upgrade to A or to wait after observing others' actions in period 1.
>
> The subsequent periods are the same as period 2. The game continues until everyone upgrades to A. The time period is denoted by t, $t = 1, 2, 3, ...$

The benefit from waiting is the information revealed about the new software package A by other individuals. The cost of waiting is the difference between the gain from A and the reservation utility.

We first investigate the relationship between the incentive to wait and private information. We prove any possible symmetric equilibrium must be monotone with respect to personal private signals. Then, we show the existence and describe characteristics of a symmetric equilibrium with the monotonicity property by backward induction in two cases: a continuous private signal space and a finite discrete private signal space.[12]

1.3.3.1 Information and Incentives

The following remark shows that if an individual gets a higher private signal, given the same history, he believes that V will be higher, i.e., the posterior distribution of V satisfies MLRP with respect to private signals.

Remark 1.1

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} \quad \textit{increasing in} \quad V \quad \forall \mu_i > \mu_i'$$

Proof. See the Appendix. ■

The benefit from upgrading to A in period t for individual i is:

$$U^A(\mu_i; h_t) = E_{V|\mu_i;h_t} V \tag{1.1}$$

The benefit from waiting in period t for individual i is:

$$U^W(\mu_i; h_t; s_{-i,t}) = \delta E_{H_{t+1}(\mu_i;h_t;s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \tag{1.2}$$

[12]Since the information disclosed through the backward induction construction process may not be monotone, the equilibrium is not necessarily unique.

where $s_{-i,t}$ represents the strategy profile of all other individuals except for individual i starting from period t; $H_{t+1}(\mu_i; h_t; s_{-i,t})$ represents the set of histories at the beginning of period $t+1$ given μ_i, h_t and $s_{-i,t}$. From the above equation, we can solve the benefit from waiting forever $\underline{U}^W = 0$. In this paper, we focus on the symmetric equilibrium.

Lemma 1.1 *Under the worst news, individuals will never upgrade to A. In our model, the worst news from period t is no one upgrading to A in period t. Under this worst news, the waiting herding phase starts in period t+1. Thus, with finite number of N individuals, the game lasts at most N periods before a herding phase starts.*

Proof. See the Appendix. ∎

The next proposition proves that for any possible symmetric equilibrium, it must be monotone with respect to personal private signals. That is, individuals with private signals indicating higher value of A have a higher incentive to upgrade.

Proposition 1.1

$$U^A(\mu_i; h_t) - U^W(\mu_i; h_t; s_{-i,t}) \quad \textit{increasing in} \quad \mu_i \quad \forall h_t; s_{-i,t}$$

Proof. See the Appendix. ∎

1.3.3.2 Symmetric Equilibrium with the Monotonicity Property

Proposition 1.2 *There exists a symmetric equilibrium with the following monotonicity property.*

(i) Case I: Continuous private signal space

The equilibrium strategy profile is a sequence of decreasing critical values: $\{\mu_t^*(h_t)\}_t$. In period t with history h_t, individuals with $\mu > \mu_t^*(h_t)$ upgrade; others wait.

Case II: Finite discrete private signal space

The equilibrium strategy profile is a sequence of decreasing critical values $\{\mu_t^*(h_t)\}_t$ and a sequence of probability of critical type $\{p_{\mu_t^*(h_t)}\}_t$. In period t with history h_t, individuals with $\mu > \mu_t^*(h_t)$ upgrade; the critical type individuals upgrade with probability $p_{\mu_t^*(h_t)}$; others wait.[13]

(ii) Large Number and Patient Individuals: *If number of individuals is large and individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed.*

Proof. See the Appendix. ■

Part (i) is from the construction in the above proposition. For part (ii), if the number of individuals is large and $\mu_1^*(h_1)$ strictly smaller than $\overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}}$ strictly greater than 0), by the Law of Large Numbers, the true value of the new software package A will be (approximately) revealed in the second period. In this case, if individuals are patient enough, then all individuals will wait in period 1 such that $\mu_1^*(h_1) = \overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}} = 0$). This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game: either $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1})$ or $\mu_t^*(h_t) \approx \mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$). Thus, at any fixed

[13] *For simplicity, $p_{\mu_t^*(h_t)} < 1$ is chosen in the construction process so that there is the possibility for the type $\mu_t^*(h_t)$ individuals to remain in the game in period $t+1$. Moreover, $\mu_t^*(h_t)$ is the highest type in period $t+1$. If a herding phase starts, all the remaining individuals in the game either upgrade ($p_{\mu_t^*(h_t)} = 1$) or wait forever ($p_{\mu_t^*(h_t)} = 0$).*

time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed.

From the above proposition, with endogenous ordering the sequence of critical values is monotone. Intuitively, in any period t, all the individuals with $\mu > \mu^*_{t-1}(h_{t-1})$ upgraded before t. In period t, we only need to consider the individuals with private signals between $\underline{\mu}$ and $\mu^*_{t-1}(h_{t-1})$. Thus, $\mu^*_t(h_t) \leq \mu^*_{t-1}(h_{t-1})$.

Upgrade herding occurs in period τ when $\mu^*_\tau(h_\tau) = \underline{\mu}$ (finite discrete signal space: $\mu^*_\tau(h_\tau) = \underline{\mu}; p_{\mu^*_\tau(h_\tau)} = 1$). All the remaining individuals upgrade to A in period τ regardless of their own private signal, and then the game ends (see figure 1.4).

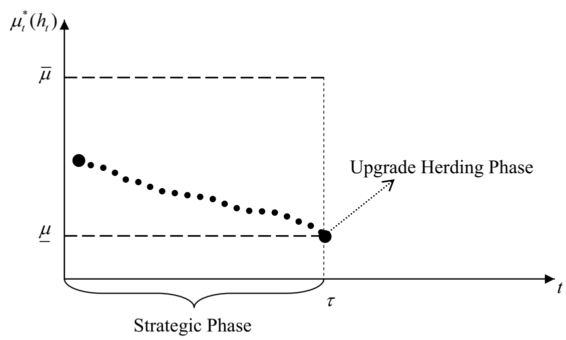

Figure 1.4: Upgrade herding with endogenous ordering

Waiting herding occurs in period τ when $\mu^*_\tau(h_\tau) = \mu^*_{\tau-1}(h_{\tau-1})$ (finite discrete signal space: $\mu^*_\tau(h_\tau) = \mu^*_{\tau-1}(h_{\tau-1}); p_{\mu^*_\tau(h_\tau)} = 0$). Since no new information is disclosed in the following periods, the game remains the same. $\mu^*_t(h_t) = \mu^*_{t-1}(h_{t-1})$ $\forall t > \tau$ (finite discrete signal space: $\mu^*_t(h_t) = \mu^*_{t-1}(h_{t-1}); p_{\mu^*_t(h_t)} = p_{\mu^*_{t-1}(h_{t-1})} = 0$ $\forall t > \tau$) (see figure 1.5).

Since the public information disclosed only needs to offset the information from individuals' actions in the last period before the waiting herding phase

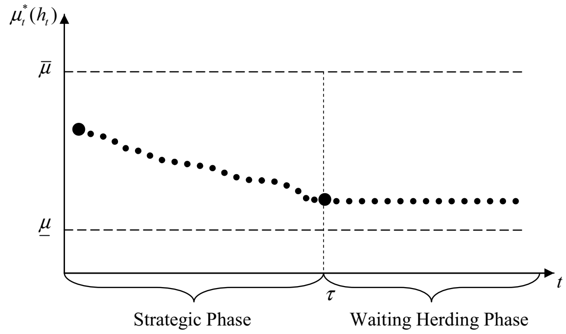

Figure 1.5: Waiting herding with endogenous ordering

starts, the waiting herding phase is not robust to the public disclosure of information. If in some period $t > \tau$ there is some public information disclosed such that $\mu^*_t(h_t) < \mu^*_{t-1}(h_{t-1})$ (finite discrete signal space: $\mu^*_t(h_t) \leq \mu^*_{t-1}(h_{t-1})$ with $p_{\mu^*_t(h_t)} > 0$), then the strategic phase starts again. However, if the game falls into the upgrade herding phase, disclosure of public information after τ will not have any effect since the upgrading herding phase only lasts one period.

1.3.4 Robustness

We summarize the results of the impact of public information disclosure on herding behavior under exogenous and endogenous ordering settings respectively in the following table.[14]

By the game construction, under the exogenous ordering setting, the game lasts exactly N periods. Disclosure of public information after period N will not have any effect. Under the endogenous ordering setting, the upgrading herding phase only lasts one period. Disclosure of public information after τ will not have any effect. In contrast, the waiting herding phase under the endogenous ordering

[14]Here, we only talk about the unexpected disclosure of public information. Section 1.4.2 investigates more variations of public information disclosure.

Table 1.2: Robustness with respect to Disclosure of Public Information

		exogenous ordering		endogenous ordering	
		upgrade herding	rejecting herding	upgrade herding	waiting herding
time of disclosure	$N \geq t > \tau$	Not Robust	Not Robust	Robust	Not Robust
of public information	$t > N$	Robust	Robust	Robust	Not Robust

setting could last forever. Disclosure of public information after τ or even N may have some effect.

1.3.5 Expected Number of Correct Choices

Proposition 1.3 *In the general model, if the number of individuals is large and individuals are patient enough, exogenous ordering is more efficient than endogenous ordering in terms of inducing a larger expected number of correct choices, if individuals can be forced to move with an exogenous order.*

Proof. From proposition 1.2, with endogenous ordering, if the number of individuals is large and individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. In contrast, in the self-decision case, each individual still utilizes his own private information and the prior probabilities. Exogenous ordering is even better since it tends to aggregate more information by forcing some of the individuals to make decisions in some given periods. Similar to result 1.1, if the number of individuals is large and individuals are patient enough, as long as the private signals and the prior probabilities are informative, exogenous ordering is more likely to induce a larger expected number of correct choices than endogenous ordering. ■

1.4 Extensions and Modifications

In this section, we first discuss the general tie-breaking rule for the exogenous ordering setting in the example from section 1.2 (with two types of individuals). Then we study the effect of the disclosure of public information for the general model.

1.4.1 General Tie-breaking Rule

In the example presented in section 1.2.1 with exogenous ordering, we follow the same tie-breaking rule in BHW. An individual indifferent between upgrading and rejecting A chooses to upgrade or to reject A with equal probability. Now, we consider the **general tie-breaking rule**: whenever individuals are indifferent between upgrading and rejecting A, type H individuals and type L individuals choose to upgrade to A with probability p_H and p_L respectively. We denote the general tie-breaking rule as $\{p_H; p_L\}$. The tie-breaking rule in BHW is a special case of the general tie-breaking rule with $\{p_H = 1/2; p_L = 1/2\}$. With the general tie-breaking rule $\{p_H; p_L\}$, the equilibrium decision rule is the same as the equilibrium decision rule in section 1.2.1 except for the tie-breaking cases.

Result 1.2 *(See the Appendix) In the example with exogenous ordering presented in section 1.2.1, with the general tie-breaking rule* $\{p_H; p_L\}$*, the expected number of correct choices is increasing in* $p_H - p_L$*. In particular,* $\{p_H = 1; p_L = 0\}$ *is the optimal tie-breaking rule in terms of inducing the maximum expected number of correct choices. Conversely,* $\{p_H = 0; p_L = 1\}$ *is the worst tie-breaking rule and the equilibrium result of exogenous ordering in this case is the same as the result of self-decision in terms of inducing the same expected number of correct choices.*

Intuitively, when type H individuals are indifferent between upgrading and rejecting A, type L individuals will for sure reject. In this case, for type H individuals, the higher p_H is, the more informative their actions are to the followers in the sense of revealing their own private signals. Conversely, for type L individuals, when they are indifferent between upgrading and rejecting A, the lower p_L is, the more informative their actions are to the followers.

1.4.2 Disclosure of Public Information

Disclosure of public information could have an influence on the strategic and herding behavior of individuals. In BHW, with exogenous ordering, initial public disclosure can make some individuals worse off ex ante. All individuals welcome public information once a herding phase has begun. Herding is delicate with respect to new information. A small amount of public information can shatter a long-lasting herd. With multiple public information disclosures, individuals eventually settle into the correct herd.

In contrast, in our general model with endogenous ordering, we distinguish the unexpected and expected disclosures of public information and announcements of future disclosure of public information in the strategic phase and the waiting herding phase respectively.

Proposition 1.4 ***(i) Disclosure of public information in the strategic phase***

If there is a disclosure of public information in the strategic phase, either unexpected or expected, all the remaining individuals welcome the new information in the ex ante sense. The announcements of future disclosure of public information will increase the individuals' incentive to wait. However, ex post some *individuals may be worse off.*

(ii) Disclosure of public information in the waiting herding phase

If there is a disclosure of public information in the waiting herding phase, either unexpected or expected, all the remaining individuals welcome the new information. And the waiting herding phase is delicate with respect to the new information. A small amount of public information can shatter a waiting herding phase. The announcements of future disclosure of public information do not have any effect on the waiting herding phase until the disclosure of public information actually happens.

(iii) Multiple disclosures of public information

Multiple disclosures of public information do not always let individuals settle into the correct herd.

Proof. See the Appendix. ■

1.5 Conclusion

In this paper, we investigate herd behavior of sequential decisions under imperfect information with one-sided commitment. We provide a framework of endogenous ordering to allow decision makers to choose the time of acting or waiting. We show the existence and characteristics of the equilibrium. We find that herd behavior under endogenous ordering is not necessarily less error prone than herd behavior under exogenous ordering due to the free-rider problem. In particular, if the number of individuals is large and individuals are patient enough, under endogenous ordering nearly all individuals are willing to wait and free-ride on others. Consequently, nearly all individuals wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information. That is to say, more “freedom” may not be better.

Another feature of the endogenous ordering framework is that in a herding phase, all the remaining individuals either act immediately or wait forever regardless of their own private signals. Thus, there exists an investment surge or collapse when herding starts. Compared with the exogenous ordering setting, disclosure of public information has a completely different impact on the strategic and herding behavior of individuals. In particular, if the game is in the upgrade herding phase, all the remaining individuals upgrade immediately and the game ends in one period. Further disclosure of public information will not have any effect.

1.6 Appendix

Proof of the Equilibrium Decision Rule with Endogenous Ordering (The Example from Section 1.2)

(i) Period 1:

In period 1, for type L individuals, the expected benefit from upgrading to A is $(\frac{1}{2} - p) < 0$. The expected benefit from waiting is greater than or equal to the benefit from waiting forever, which equals zero. Therefore, type L individuals will wait for sure in period 1. For type H individuals, the expected benefit from upgrading to A is $(p - \frac{1}{2}) > 0$. If no one else upgrades to A, the expected benefit from waiting is equal to the benefit from waiting forever, which equals zero. Thus, a type H individual will upgrade to A if no one else upgrades.

Let us check the condition that a type H individual i will upgrade to A for sure in period 1 when all other type H individuals upgrade to A for sure in period

1. The expected benefit from upgrading to A is $(p - \frac{1}{2})$. Since all other type H individuals upgrade to A for sure in period 1, all of the possible information is disclosed for individual i in period 2. If the number of individuals upgrading to A in period 1 is greater than or equal to $\lceil \frac{N-1}{2} \rceil$, where $\lceil \frac{N-1}{2} \rceil$ is the smallest integer greater than or equal to $\frac{N-1}{2}$, in period 2 individual i will upgrade to A. Thus, the expected benefit from waiting is

$$\delta \sum_{j=\lceil \frac{N-1}{2} \rceil}^{N-1} Prob(\text{j upgrade in period 1}|\mu_i = H)[Prob(V = 1/2|\text{j+1 H in N}) - 1/2]$$

where

$$Prob(\text{j upgrade in period 1}|\mu_i = H) = \binom{N-1}{j}[p^{j+1}(1-p)^{N-1-j} + (1-p)^{j+1}p^{N-1-j}]$$

$$Prob(V = 1/2|\text{j+1 H in N}) = \frac{p^{2j+2-N}}{p^{2j+2-N} + (1-p)^{2j+2-N}}$$

Let the expected benefit from upgrading to A equal the expected benefit from waiting. We get

$$\delta^*(N,p) = \frac{p - \frac{1}{2}}{\sum_{j=\lceil \frac{N-1}{2} \rceil}^{N-1} \binom{N-1}{j}[p^{j+1}(1-p)^{N-1-j} + (1-p)^{j+1}p^{N-1-j}][\frac{p^{2j+2-N}}{p^{2j+2-N}+(1-p)^{2j+2-N}} - 1/2]}$$

Clearly, $\delta^*(N,p)$ is an increasing function of p and decreasing function of N.

Figure 1.6 illustrates the locus of $\delta^*(N, p = 0.6)$ decreasing in N. The upper right area is the mixed strategy area of type H individuals, in which type H individuals upgrade to A with some probability $0 < p_{H,1} < 1$. The lower left area is the pure strategy area that type H individuals will for sure upgrade to A in period 1. The intuition is that as the number of individuals increases, everyone has a higher incentive to wait. To induce type H individuals to upgrade to A with probability one, the discount factor should be low.

Figure 1.7 illustrates the locus of $\delta^*(N,p)$ shifting up as p increases. As the precision of signals p increases, type H individuals have a higher incentive to upgrade to A. Thus, the pure strategy area of type H individuals becomes larger as p increases.

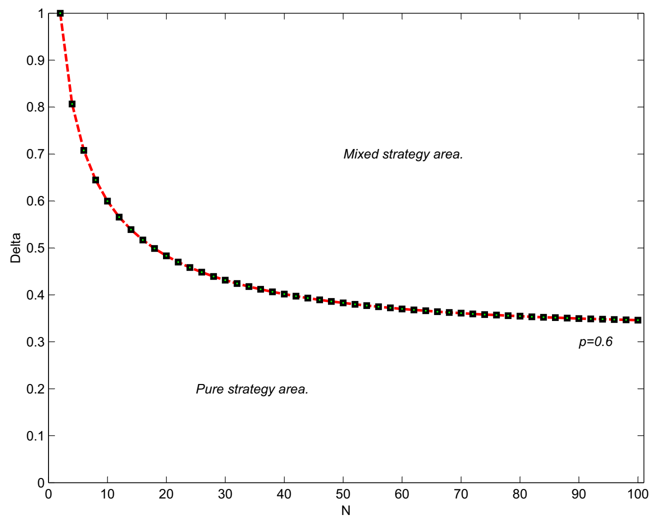

Figure 1.6: The Example from Section 1.2 – Endogenous Ordering

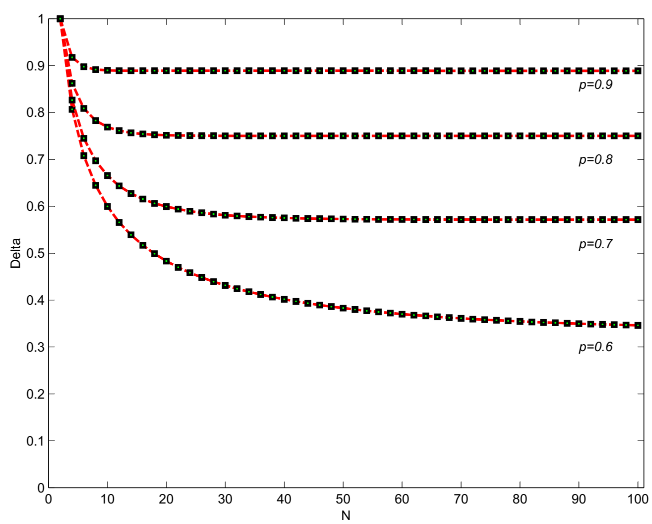

Figure 1.7: The Example from Section 1.2 – Endogenous Ordering

For the symmetric equilibrium, in period 1 type L individuals will wait to see type H's action. Type H individuals will upgrade to A for sure if $\delta \leq \delta^*(N, p)$. Otherwise, type H individuals will upgrade to A with some probability $0 < p_{H,1} <$

1.

Similar to the proof of Proposition 1 in Chamley and Gale (1994), as $p_{H,1}$ and N increases, information in period 2 is more informative in the sense of Blackwell. From Blackwell's theorem, the expected benefit from waiting increases. As δ increases, the expected benefit from waiting increases. As p decreases, the expected benefit from upgrading to A decreases.

To keep type H individuals indifferent between upgrading to A immediately and waiting, we must have $p_{H,1}$ decreasing in δ and N, and increasing in p. Figure 1.8 illustrates the locus of $p_{H,1}$ when $N = 3$.

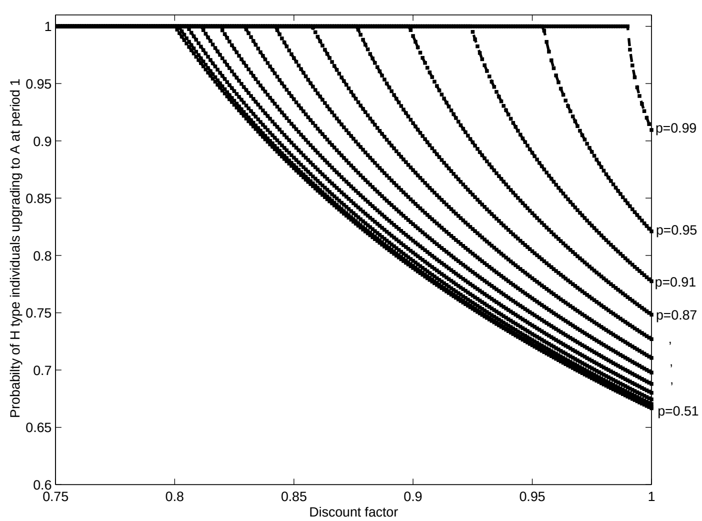

Figure 1.8: The Example from Section 1.2 – Endogenous Ordering: N=3

(ii) Large Number of Individuals:

If the number of individuals is large, from the above proof we can easily check that $\delta^*(p) = \lim_{N\to\infty} \delta^*(N,p) = \frac{2p-1}{p}$. In another way, we may find $\delta^*(p)$ under the condition that a type H individual i is indifferent between upgrading and waiting when he knows the true value of the new software package A will be revealed in the second period. That is $\delta^*(p)$ is the solution of the following equation.

$$p - 1/2 = \delta E_{V|\mu_i;h_1}[max\{V; 0\}] = \delta[1/2p + 0(1-p)]$$

If $\delta \leq \delta^*(p)$, regardless of the number of individuals in the game, type H individuals will for sure upgrade in period 1, since $\delta^*(p) \leq \delta^*(N,p)$. If $p_{H,1}$ is strictly greater than 0, by the Law of Large Numbers, the true value of the new software package A will be (approximately) revealed in the second period. In this case, if $\delta > \delta^*(p)$, then all individuals will wait in period 1 such that $p_{H,1}$ is equal to 0. This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game. The probability of type H individuals upgrading to A in period t, which is denoted by $p_{H,t}$, is equal to 0 or approximately equal to 0. Otherwise, similarly as in period 1, if there exists some finite period T such that $p_{H,T}$ strictly greater than 0, then all individuals will wait till period $T+1$. This means $p_{H,t} = 0 \; \forall t \leq T$. That is a contradiction. Consequently, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed. ■

Proof of Result 1.1

(Complete information) With complete information, the true value of A is known. Everyone makes the correct choice, which means $X_{CI}(N)/N = 1$. Certainly, this is the upper bound of $X(N)/N$.

(Social planner) If there is a social planner who can gather the private information from all the individuals, then based on all the private signals and the prior probabilities,

$$X_{SP}(N)/N = \sum_{j>N/2}^{N} \binom{N}{j} p^j (1-p)^{N-j} + \mathbf{1}_{\{\text{N is even}\}} \left[\binom{N}{N/2} p^{N/2} (1-p)^{N/2} \frac{1}{2} \right]$$

where $\mathbf{1}_{\{\text{N is even}\}}$ is the indicator function; if N is even, $\mathbf{1}_{\{\text{N is even}\}} = 1$; otherwise, $\mathbf{1}_{\{\text{N is even}\}} = 0$.

(Self-decision) If there are no interactions among the individuals, each individual makes a self-decision using his own private signal and the prior probabilities. Then based on the precision of private signals, $X_{SD}(N)/N = p$.

(Exogenous ordering) With exogenous ordering, according to the equilibrium decision rule in section 1.2.1, we have

$$\begin{cases} X_{EX}(1)/1 = p \\ X_{EX}(2)/2 = p \\ X_{EX}(N)/N = p^2 + p(1-p)\left\{\frac{N-2}{N}\left[\frac{1}{N-2}X_{EX}(N-2) - \frac{1}{2}\right] + 1\right\} \forall N \geq 3 \end{cases}$$

We can easily check that $X_{EX}(3)/3 > p$ and $X_{EX}(4)/4 > p$. Then plugging back to the above formula and by induction, we have $X_{EX}(N)/N > X_{SD}(N)/N = p$, $\forall N \geq 3$. As $N \to \infty$, $X_{EX}(N)/N \to p\frac{1/2(1+p)}{1-p+p^2}$, where $\frac{1/2(1+p)}{1-p+p^2} > 1$.

(Endogenous ordering)

With endogenous ordering, $X_{EN}(1)/1 = p$; $X_{EN}(2)/2 = p$. The results are equivalent to the cases of self-decision and exogenous ordering with 1 or 2 individuals respectively.

(i) Impatient Individuals

According to the equilibrium decision rule in section 1.2.2, if $\delta \leq \delta^*(p) \leq \delta^*(N,p)$, type H individuals will for sure upgrade in period 1 and type L individuals will wait to see type H's action. In period 2, all the possible information is disclosed. Thus, conditional on $V = 1/2$,

$$X_{EN|V=1/2}(N)/N = \sum_{j>N/2}^{N} \binom{N}{j} p^j (1-p)^{N-j} + \mathbf{1}_{\{\text{N is even}\}} \left[\binom{N}{N/2} p^{N/2} (1-p)^{N/2} 3/4 \right] + \sum_{j=0}^{j<N/2} \binom{N}{j} p^j (1-p)^{N-j} j/N$$

Conditional on $V = -1/2$,

$$X_{EN|V=-1/2}(N)/N = \mathbf{1}_{\{\text{N is even}\}} \left[\binom{N}{N/2} (1-p)^{N/2} p^{N/2} 1/4 \right] + \sum_{j=0}^{j<N/2} \binom{N}{j} (1-p)^j p^{N-j} (N-j)/N$$

Unconditional average expected number of corrected choices,

$$X_{EN}(N)/N = \frac{1}{2} X_{EN|V=1/2}(N)/N + \frac{1}{2} X_{EN|V=-1/2}(N)/N$$

Using the exhaustion method (Matlab simulation), we can check that $X_{SP}(N)/N > X_{EN}(N)/N > X_{EX}(N)/N$ for $N \geq 3$ and not too large. For N large but still less than N^*, $X_{EN|V=1/2}(N)/N$ converges to 1 and $X_{EN|V=-1/2}(N)/N$ to p. Then $X_{EN}(N)/N$ converges to $p\frac{1+p}{2p}$. $\frac{1+p}{2p} > \frac{1/2(1+p)}{1-p+p^2}$ implies $X_{EN}(N)/N > X_{EX}(N)/N$ for N large.

(ii) Patient Individuals

If $\delta > \delta^*(p)$, by continuity, there exists an N^* such that $\delta = \delta^*(N^*, p)$, since $\delta^*(N, p)$ is decreasing in N. If $N \leq N^*$, then $\delta \leq \delta^*(N, p)$. Similar as the proof of Impatient Individuals case above, in period 1, type H individuals will upgrade to A for sure and type L individuals will wait to see type H's action. In period 2, all the possible information is disclosed. We have $X_{EN}(N)/N > X_{EX}(N)/N$ for $N \leq N^*$.

If N is large enough, according to the equilibrium decision rule in section 1.2.2, for any finite period t, $p_{H,t}$ is either zero or approximately equal to zero.

At any fixed time, nearly all individuals wait due to the negligible information disclosed. Thus, there exists an N^{**} such that for all $N > N^{**}$, $X_{EN|V=1/2}(N)/N$ converges to zero and $X_{EN|V=-1/2}(N)/N$ to one. Then $X_{EN}(N)/N$ converges to 1/2, which is less than $X_{SD}(N)/N = p$. ■

Proof of Remark 1

Since it has been assumed that the private signals μ are independent conditional on V, we have

$$f(V|\mu_i, h_t) = \frac{f(\mu_i, h_t|V)f_0(V)}{f(\mu_i, h_t)} = \frac{f(\mu_i|V)f(h_t|V)f_0(V)}{f(\mu_i, h_t)}$$

$$f(V|\mu_i', h_t) = \frac{f(\mu_i', h_t|V)f_0(V)}{f(\mu_i', h_t)} = \frac{f(\mu_i'|V)f(h_t|V)f_0(V)}{f(\mu_i', h_t)}$$

This implies

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} = \frac{\frac{f(\mu_i|V)f(h_t|V)f_0(V)}{f(\mu_i,h_t)}}{\frac{f(\mu_i'|V)f(h_t|V)f_0(V)}{f(\mu_i',h_t)}} = \frac{f(\mu_i|V)}{f(\mu_i'|V)}\frac{f(\mu_i', h_t)}{f(\mu_i, h_t)}$$

Then $\forall V_1 > V_2$,

$$\frac{\frac{f(V_1|\mu_i,h_t)}{f(V_1|\mu_i',h_t)}}{\frac{f(V_2|\mu_i,h_t)}{f(V_2|\mu_i',h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i'|V_1)}\frac{f(\mu_i',h_t)}{f(\mu_i,h_t)}}{\frac{f(\mu_i|V_2)}{f(\mu_i'|V_2)}\frac{f(\mu_i',h_t)}{f(\mu_i,h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)}}{\frac{f(\mu_i'|V_1)}{f(\mu_i'|V_2)}}$$

By Assumption 1,

$$\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)} \quad \text{increasing in} \quad \mu_i \quad \forall V_1 > V_2$$

So, we have $\forall \mu_i > \mu_i'$

$$\frac{\frac{f(V_1|\mu_i,h_t)}{f(V_1|\mu_i',h_t)}}{\frac{f(V_2|\mu_i,h_t)}{f(V_2|\mu_i',h_t)}} = \frac{\frac{f(\mu_i|V_1)}{f(\mu_i|V_2)}}{\frac{f(\mu_i'|V_1)}{f(\mu_i'|V_2)}} \geq 1$$

which means

$$\frac{f(V|\mu_i, h_t)}{f(V|\mu_i', h_t)} \quad \text{increasing in} \quad V \quad \forall \mu_i > \mu_i'$$

■

Proof of Lemma 1.1

If in period t individual i chooses to wait, then $U^A(\mu_i; h_t) \leq U^W(\mu_i; h_t; s_{-i,t})$.

By the Martingale property,

$$U^A(\mu_i; h_t) = E_{H_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1})$$

The set of histories $H_{t+1}(\mu_i; h_t; s_{-i,t})$ can be decomposed into two disjoint sets: $H^A_{t+1}(\mu_i; h_t; s_{-i,t})$ and $H^W_{t+1}(\mu_i; h_t; s_{-i,t})$, where $H^A_{t+1}(\mu_i; h_t; s_{-i,t})$ is the set of histories in period $t+1$ in which individual i will upgrade to A according to some strategy s_i of individual i; $H^W_{t+1}(\mu_i; h_t; s_{-i,t})$ is the set of histories in period $t+1$ in which individual i will wait according to some strategy s_i of individual i. Then we have

$$U^A(\mu_i; h_t) = E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1})$$

By equation 1.2,

$$\begin{aligned} U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H_{t+1}(\mu_i; h_t; s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \\ &= \delta[E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i; h_t; s_{-i,t})} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \end{aligned}$$

Suppose under the worst news from period t individual i still upgrades in period $t+1$. Then he will for sure upgrade in period $t+1$, which means $H^W_{t+1}(\mu_i; h_t; s_{-i,t}) = \emptyset$.

Back to the above equations, we have

$$\begin{aligned} U^A(\mu_i; h_t) &= E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) \\ U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H^A_{t+1}(\mu_i; h_t; s_{-i,t})} U^A(\mu_i; h_{t+1}) \end{aligned}$$

Since $H^W_{t+1}(\mu_i; h_t; s_{-i,t}) = \emptyset$, $U^W(\mu_i; h_t; s_{-i,t}) > \underline{U}^W = 0$. We have $U^A(\mu_i; h_t) > U^W(\mu_i; h_t; s_{-i,t})$. This is a contradiction.

In our model, the worst news in period t is no one upgrades. Under this worst news, the waiting herding phase starts in period $t+1$. To keep the upgrade going, at least one individual must upgrade to A in each period. Thus, with finite number of N individuals, the game lasts at most N periods before a herding phase starts. ■

Proof of Proposition 1.1

By Remark 1, $f(V|\mu, h_t)$ satisfies MLRP with respect to μ. According to Landsberger and Meilijson (1990), $F(V|\mu_i, h_t)$ first order stochastic dominates (FOSD) $F(V|\mu_i', h_t)$ for any $\mu_i > \mu_i'$. So, $U^A(\mu_i; h_t) \geq U^A(\mu_i'; h_t)$ for any h_t.

Similar to the proof of Lemma 1.1, by the Martingale property,

$$U^A(\mu_i; h_t) = E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1})$$

$$\begin{aligned} U^W(\mu_i; h_t; s_{-i,t}) &= \delta E_{H_{t+1}(\mu_i;h_t;s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}] \\ &= \delta[E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \end{aligned}$$

Thus, for any non-negative integer j

$$\begin{aligned} U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) &= (1-\delta^{j+1}) E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) \\ &+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})}[U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \end{aligned} \tag{1.3}$$

Let us check the incentives of waiting and upgrading for individual i who has a lower private signal $\mu_i' < \mu_i$. Similarly, we have

$$\begin{aligned} U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t}) &= (1-\delta^{j+1}) E_{H^A_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i'; h_{t+1}) \\ &+ E_{H^W_{t+1}(\mu_i';h_t;s_{-i,t})}[U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^W(\mu_i'; h_{t+1}; s_{-i,t+1})] \end{aligned} \tag{1.4}$$

By Lemma 1.1, the game lasts at most N periods before a herding phase starts. Suppose either an upgrading or a waiting herding phase starts in period $T \leq N$, which means no one will upgrade to A after period T given history h_T and strategy profile $(s_{i,T}, s_{-i,T})$. With a herding phase starting in period T, no more new information is disclosed in period $T+1$, which means $U^W(\mu_i; h_T; s_{-i,T}) = U^W(\mu_i'; h_T; s_{-i,T}) = 0$. Thus, in period T,

$$U^A(\mu_i; h_T) - \delta^j U^W(\mu_i; h_T; s_{-i,T}) \geq U^A(\mu_i'; h_T) - \delta^j U^W(\mu_i'; h_T; s_{-i,T})$$

Back to period $T-1$, since herding starts in period T, $H_T^A(\mu_i; h_{T-1}; s_{-i,T-1}) = H_T^A(\mu_i'; h_{T-1}; s_{-i,T-1})$ and $H_T^W(\mu_i; h_{T-1}; s_{-i,T-1}) = H_T^W(\mu_i'; h_{T-1}; s_{-i,T-1})$. By equation 1.3 and 1.4,

$$U^A(\mu_i; h_{T-1}) - \delta^j U^W(\mu_i; h_{T-1}; s_{-i,T-1}) \geq U^A(\mu_i'; h_{T-1}) - \delta^j U^W(\mu_i'; h_{T-1}; s_{-i,T-1})$$

When $j = 0$, the above formula implies that individuals with private signals indicating higher value of the new software package A have a higher incentive to upgrade given the same public history in period $T-1$. That is to say,

$$H_{T-1}^A(\mu_i; h_{T-2}; s_{-i,T-2}) \supseteq H_{T-1}^A(\mu_i'; h_{T-2}; s_{-i,T-2})$$
$$H_{T-1}^W(\mu_i; h_{T-2}; s_{-i,T-2}) \subseteq H_{T-1}^W(\mu_i'; h_{T-2}; s_{-i,T-2})$$

Back to period $T-2$, by equation 1.3 and 1.4,

$$\begin{aligned} U^A(\mu_i; h_{T-2}) - \delta^j U^W(\mu_i; h_{T-2}; s_{-i,T-2}) &= (1-\delta^{j+1}) E_{H_{T-1}^A(\mu_i'; h_{T-2}; s_{-i,T-2})} U^A(\mu_i; h_{T-1}) \\ &+ (1-\delta^{j+1}) E_{H_{T-1}^A(\mu_i; h_{T-2}; s_{-i,T-2}) \cap H_{T-1}^W(\mu_i'; h_{T-2}; s_{-i,T-2})} U^A(\mu_i; h_{T-1}) \\ &+ E_{H_{T-1}^W(\mu_i; h_{T-2}; s_{-i,T-2})} [U^A(\mu_i; h_{T-1}) - \delta^{j+1} U^W(\mu_i; h_{T-1}; s_{-i,T-1})] \end{aligned}$$

$$\begin{aligned} U^A(\mu_i'; h_{T-2}) - \delta^j U^W(\mu_i'; h_{T-2}; s_{-i,T-2}) &= (1-\delta^{j+1}) E_{H_{T-1}^A(\mu_i'; h_{T-2}; s_{-i,T-2})} U^A(\mu_i'; h_{T-1}) \\ &+ E_{H_{T-1}^A(\mu_i; h_{T-2}; s_{-i,T-2}) \cap H_{T-1}^W(\mu_i'; h_{T-2}; s_{-i,T-2})} [U^A(\mu_i'; h_{T-1}) - \delta^{j+1} U^W(\mu_i'; h_{T-1}; s_{-i,T-1})] \\ &+ E_{H_{T-1}^W(\mu_i; h_{T-2}; s_{-i,T-2})} [U^A(\mu_i'; h_{T-1}) - \delta^{j+1} U^W(\mu_i'; h_{T-1}; s_{-i,T-1})] \end{aligned}$$

For $h_{T-1} \in [H_{T-1}^A(\mu_i; h_{T-2}; s_{-i,T-2}) \cap H_{T-1}^W(\mu_i'; h_{T-2}; s_{-i,T-2})]$,

$$U^W(\mu_i'; h_{T-1}; s_{-i,T-1}) \geq U^A(\mu_i'; h_{T-1})$$

which implies $U^A(\mu_i'; h_{T-1}) - \delta^{j+1} U^W(\mu_i'; h_{T-1}; s_{-i,T-1}) \leq (1-\delta^{j+1}) U^A(\mu_i'; h_{T-1}) \leq (1-\delta^{j+1}) U^A(\mu_i; h_{T-1})$. Thus,

$$U^A(\mu_i; h_{T-2}) - \delta^j U^W(\mu_i; h_{T-2}; s_{-i,T-2}) \geq U^A(\mu_i'; h_{T-2}) - \delta^j U^W(\mu_i'; h_{T-2}; s_{-i,T-2})$$

And so on, for any $t \leq T$

$$U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) \geq U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t})$$

Let $j = 0$. We are done. ∎

Proof of Proposition 1.2

(i) Case I: Continuous private signal space

Let $\mathcal{G}_n(h_t)$ represent the subgame starting from period t with history h_t, where n is the number of individuals remaining in this subgame.[15] Use backward induction.

Step 1 Start from the subgame with only one individual, $\mathcal{G}_1(h_t)$. We can find a critical value $\mu_t^*(h_t)$ which is the solution of $U^A(\mu_t^*(h_t); h_t) = 0$. An individual with $\mu > \mu_t^*(h_t)$ upgrades; otherwise, he waits forever.

Step 2 Now consider the subgame with two individuals $\mathcal{G}_2(h_t)$, by Lemma 1.1, this subgame lasts at most 2 periods. In period $t+1$, there are three possible cases: (1) there are still two individuals remaining in the game (waiting herding starts); (2) there is only one individual remaining in the game ($\mathcal{G}_1(h_{t+1})$); (3) there is no one remaining in the game (game ends). By Proposition 1.1, for any symmetric equilibrium, individuals with private signals indicating higher value of A have a higher incentive to upgrade.

[15] n must be compatible with h_t.

We can find a critical value $\mu_t^*(h_t)$ which is a function of $\mu_{t+1}^*(h_{t+1})$ in the subsequent subgame $\mathcal{G}_1(h_{t+1})$. Individuals with $\mu > \mu_t^*(h_t)$ upgrade; otherwise, they wait.

⋮

⋮

Step N Continue to the subgame with N individuals $\mathcal{G}_N(h_t)$, by Lemma 1.1, this subgame lasts at most N periods. Similarly, there are $N+1$ possible cases: (1) there are still N individuals remaining in the game (waiting herding starts); (2) there are $N-1$ individuals remaining in the game ($\mathcal{G}_{N-1}(h_{t+1})$);...; ($N$) there is only 1 individual remaining in the game ($\mathcal{G}_1(h_{t+1})$); ($N+1$) there is no one remaining in the game (game ends). We can find a critical value $\mu_t^*(h_t)$ which is a function of $\mu_{t+1}^*(h_{t+1})$ in the subsequent subgames $\mathcal{G}_{N-1}(h_{t+1}), \ldots, \mathcal{G}_1(h_{t+1})$. Individuals with $\mu > \mu_t^*(h_t)$ upgrade; otherwise, they wait.

We can see in the final Step N if we replace h_t with h_1 then $\mathcal{G}_N(h_t)$ is the original game.

Case II: Finite discrete private signal space

Denote the private signal space by $\{\mu_1, \mu_2, \ldots, \mu_K\}$, where $\mu_1 < \mu_2 < \ldots < \mu_K$. The strategy profile starting from period t, $s_t = \{P_\tau\}_{\tau=t}^{\infty}$, where $P_\tau = \{p_{\mu_k,\tau}\}_{k=1}^{K}$ and $p_{\mu_k,\tau}$ represents the probability of type μ_k upgrading to A in period τ. For μ_i, $U^A(\mu_i; h_t) - U^W(\mu_i; h_t; \{P_\tau\}_{\tau=t}^{\infty})$ is continuous in $p_{\mu_k,\tau}$ $\forall \mu_k, \tau$. Let $\mathcal{G}_{M,n}(h_t)$ represent the subgame starting from period t with history h_t, where M and n are the set of possible types and the number of individuals remaining in this subgame respectively.[16]

[16] M, n must be compatible with h_t.

By Proposition 1.1, for any symmetric equilibrium, individuals with private signals indicating higher value of A have a higher incentive to upgrade. Thus, with a finite number of individuals and a finite number of individual types, backward induction can be used to construct the symmetric equilibrium through the following steps.

Step 1.1 Start from the subgame $\mathcal{G}_{\{\mu_1\},1}(h_t)$ with only one individual whose private signal is μ_1. That is, $M = \{\mu_1\}$. Since all the information is disclosed, $U^W(\mu_1; h_t; \{p_{\mu_1,t}\}) = \underline{U}^W = 0$. There are three possible cases:

1.1.1 If $U^A(\mu_1; h_t) < 0$, $\{p_{\mu_1,t} = 0\}$ is the equilibrium strategy profile in period t. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

1.1.2 If $U^A(\mu_1; h_t) = 0$, $\{0 \leq p_{\mu_1,t} \leq 1\}$ will be the equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in period $t+1$ is the same as the period t game $\mathcal{G}_{\{\mu_1\},1}(h_t)$, since $h_{t+1} = h_t$.

1.1.3 If $U^A(\mu_1; h_t) > 0$, $\{p_{\mu_1,t} = 1\}$ is the equilibrium strategy profile and game ends.

⋮

Step 1.N Consider the subgame $\mathcal{G}_{\{\mu_1\},N}(h_t)$ with N individuals whose private signals are μ_1. Since all the information is disclosed, $U^W(\mu_1; h_t; \{p_{\mu_1,t}\}) = \underline{U}^W = 0$. Same as the subgame $\mathcal{G}_{\{\mu_1\},1}(h_t)$, there are three possible cases:

1.N.1 If $U^A(\mu_1; h_t) < 0$, $\{p_{\mu_1,t} = 0\}$ is the equilibrium strategy profile in period t. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

1.N.2 If $U^A(\mu_1; h_t) = 0$, $\{0 \leq p_{\mu_1,t} \leq 1\}$ will be the equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in period $t+1$ is $\mathcal{G}_{\{\mu_1\},n}(h_{t+1})$, where $0 < n < N$. Note, $h_\tau = h_t$, $\forall \tau > t$.

1.N.3 If $U^A(\mu_1; h_t) > 0$, $\{p_{\mu_1,t} = 1\}$ is the equilibrium strategy profile and game ends.

Step 2.1 Now consider the subgame $\mathcal{G}_{\{\mu_1,\mu_2\},1}(h_t)$ with only one individual whose type belongs to μ_1, μ_2. That is, $M = \{\mu_1, \mu_2\}$. There are three possible cases:

2.1.1 If $0 \geq U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0\}$ is an equilibrium strategy profile in period t. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

2.1.2 If $U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait in period t. Since all the information is disclosed, $U^W(\mu_2; h_t; \{p_{\mu_1,t}, p_{\mu_2,t}\}) = U^W(\mu_1; h_t; \{p_{\mu_1,t}, p_{\mu_2,t}\}) = \underline{U}^W = 0$. Thus, $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$ is an equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

2.1.3 If $U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1\}$ is an equilibrium strategy profile and game ends.

$\vdots$

Step 2.N Now consider the subgame $\mathcal{G}_{\{\mu_1,\mu_2\},N}(h_t)$ with N individuals whose types belong to μ_1, μ_2. That is, $M = \{\mu_1, \mu_2\}$. There are three possible cases:

2.N.1 If $0 \geq U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0\}$ is an equilibrium strategy profile in period t. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

2.N.2 If $U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait in period t. Consider the strategy profile in period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$. There are two possible cases:

2.N.2.1 If $U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}) \geq 0 > U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\})$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$ is an equilibrium strategy profile in period t. The continuation game in period $t+1$ is $\mathcal{G}_{\{\mu_1\},n}(h_{t+1})$ if the game does not end in period t, where $0 < n < N$.[17]

2.N.2.2 If $0 > U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}) \geq U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\})$, then decreasing $p_{\mu_2,t}$ till $0 = U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t}\}) \geq U^A(\mu_1; h_t) - U^W(\mu_1; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t}\})$ by continuity. The solution $\{p_{\mu_1,t} = 0, p_{\mu_2,t}\}$ to the above formula is an equilibrium strategy profile in period t. The continuation game in period $t+1$ is $\mathcal{G}_{\{\mu_1,\mu_2\},n}(h_{t+1})$ if the game does not end in period t, where $0 < n < N$.

2.N.3 If $U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1\}$ is an equilibrium strategy profile and game ends.

$\vdots$

$\vdots$

[17]Note, since the benefit from waiting is derived from the continuation game, we must construct the entire scheme of the continuation game first, then find out if the conjectured strategy profile $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1\}$ is indeed an equilibrium in period t. Same logic applies to the following proof.

Step K.1 Continue to the subgame $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\},1}(h_t)$ with only one individual whose type belongs to $\mu_1, \mu_2, \ldots, \mu_K$. That is, $M = \{\mu_1, \mu_2, \ldots, \mu_K\}$. There are $K+1$ possible cases:

K.1.1 If $0 \geq U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_K,t} = 0\}$ is an equilibrium. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

K.1.2 If $U^A(\mu_K; h_t) > 0 \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t)$, then $\mu_1, \mu_2, \ldots, \mu_{K-1}$ types will for sure wait in period t. Since all the information is disclosed, $U^W(\mu_K; h_t; \{p_{\mu_1,t}, p_{\mu_2,t}, \ldots, p_{\mu_{K-1},t}, p_{\mu_K,t}\}) = \ldots = U^W(\mu_1; h_t; \{p_{\mu_1,t}, p_{\mu_2,t}, \ldots, p_{\mu_{K-1},t}, p_{\mu_K,t}\}) = \underline{U}^W = 0$. Thus, $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

$\vdots$

K.1.K If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait in period t. Same logic, $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1, \ldots, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

K.1.K+1 If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1, \ldots, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile and game ends.

⋮

Step K.N Continue to the subgame $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\},N}(h_t)$ with N individuals whose types belong to $\mu_1, \mu_2, \ldots, \mu_K$. That is, $M = \{\mu_1, \mu_2, \ldots, \mu_K\}$. There are $K + 1$ possible cases:

K.N.1 If $0 \geq U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_K,t} = 0\}$ is an equilibrium. The continuation game in the following periods is the same as the period t game since $h_\tau = h_t$, $\forall \tau > t$.

K.N.2 If $U^A(\mu_K; h_t) > 0 \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t)$, then $\mu_1, \mu_2, \ldots,$ μ_{K-1} types will for sure wait in period t. Consider the strategy profile in period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}$. There are two possible cases:

K.N.2.1 If $U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}) \geq 0 > U^A(\mu_{K-1}; h_t) - U^W(\mu_{K-1}; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$, then $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in period $t + 1$ is $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_{K-1}\},n}(h_{t+1})$, where $0 < n < N$.

K.N.2.2 If $0 > U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots,$ $p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$, then decreasing $p_{\mu_K,t}$ till $0 = U^A(\mu_K; h_t) - U^W(\mu_K; h_t; \{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t} = 1\})$ by continuity. The solution $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 0, \ldots, p_{\mu_{K-1},t} = 0, p_{\mu_K,t}\}$ to the above formula is an equilibrium strategy profile in period t. If the game does not end in period t, the continuation game in period $t + 1$ is $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\},n}(h_{t+1})$, where $0 < n < N$.

$\vdots$

K.N.K If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_2; h_t) > 0 > U^A(\mu_1; h_t)$, then μ_1 type will for sure wait in period t. Consider the strategy profile in period t: $\{p_{\mu_1,t} = 0, p_{\mu_2,t} = 1, \ldots, p_{\mu_K,t} = 1\}$. There are K possible cases. Check if this strategy profile is an equilibrium strategy profile in period t. If not, decrease $p_{\mu_2,t}$ from 1 to 0. Then decrease $p_{\mu_3,t}$ from 1 to 0. And so on, decrease $p_{\mu_K,t}$ from 1 to 0. Eventually, by continuity, we will find a symmetric equilibrium for subgame $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\},N}(h_t)$.

K.N.K+1 If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t) \geq 0$, then $\{p_{\mu_1,t} = 1, p_{\mu_2,t} = 1, \ldots, p_{\mu_K,t} = 1\}$ is an equilibrium strategy profile and game ends.

We can see at the final Step K if we replace h_t with h_1 then $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\},N}(h_1)$ is the original game.

(ii) Large Number and Patient Individuals:

If the number of individuals is large and $\mu_1^*(h_1)$ strictly smaller than $\overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}}$ strictly greater than 0), by the Law of Large Numbers, the true value of the new software package A will be (approximately) revealed in the second period. In this case, if individuals are patient enough, then all individuals will wait in period 1 such that $\mu_1^*(h_1) = \overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}} = 0$). This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game: either $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1})$ or $\mu_t^*(h_t) \approx \mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}} = 0$ or $p_{\overline{\mu}} \approx 0$). Otherwise, if there exists some finite period T such that $\mu_t^*(h_t)$ strictly smaller than $\mu_{t-1}^*(h_{t-1})$ (finite discrete private signal space: $p_{\overline{\mu}}$ strictly greater than 0), then all individuals will wait until period $T+1$ since they are patient enough. This means the probability for all types of individuals upgrading to A is equal to zero $\forall t \leq T$. That is a contradiction. Thus, at any fixed time, there is a negligible proportion of individuals upgrading to A and so is the information disclosed. ■

Proof of Result 1.2

According to the equilibrium decision rule in section 1.2.1, with the general tie-breaking rule $\{p_H; p_L\}$, we have $X_{EX}(1)/1 = X_{EX}(2)/2 = p$ and $\forall N \geq 3$

$$X_{EX}(N)/N = p^2 + p(1-p)\left\{\frac{N-2}{N}(1+p_H-p_L)\left[\frac{1}{N-2}X_{EX}(N-2)-\frac{1}{2}\right]+1\right\}$$

We can easily check that $X_{EX}(3)/3 \geq p$ and $X_{EX}(4)/4 \geq p$. Then plugging back to the above formula and by induction, we have $X_{EX}(N)/N \geq p$, $\forall N$. As $N \to \infty$, $X_{EX}(N)/N \to \frac{p^2+1/2p(1-p)(1-(p_H-p_L))}{1-p(1-p)(1+(p_H-p_L))}$.

Since $\forall N$, $X_{EX}(N)/N \geq p > \frac{1}{2}$, from the above formula, we can see $X_{EX}(N)/N$ increasing in $p_H - p_L$. In particular, $X_{EX}(N)/N$ achieves its maximum when $\{p_H = 1; p_L = 0\}$. $X_{EX}(N)/N$ achieves its minimum p when $\{p_H = 0; p_L = 1\}$. In other words, with the tie-breaking rule $\{p_H = 0; p_L = 1\}$, the equilibrium result of exogenous ordering is the same as the result of self-decision in terms of inducing the same expected number of correct choices. ■

Proof of Proposition 1.4

(i) Disclosure of public information in the strategic phase

Suppose there is a disclosure of public information at the beginning of period t which belongs to the strategic phase, either unexpected or expected. In the ex ante sense, without the new information, the expected payoff for individual i is

$$max\{U^A(\mu_i; h_t); U^W(\mu_i; h_t; s_{-i,t})\}$$

With the new information, the expected payoff for individual i is

$$E_{\tilde{H}_t^A(\mu_i;h_t;s_{-i,t})}U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}_t^W(\mu_i;h_t;s_{-i,t})}U^W(\mu_i; \tilde{h}_t)$$

where $\tilde{H}_t^A(\mu_i; h_t; s_{-i,t})$ is the set of histories in period t in which with the new information individual i will upgrade according to some strategy s_i. $\tilde{H}_t^W(\mu_i; h_t; s_{-i,t})$ is the set of histories in period t in which with the new information individual i will wait according to some strategy s_i.

Following the same logic as presented in the proof of Proposition 1.1, by the Martingale property,

$$\begin{aligned} U^A(\mu_i; h_t) &= E_{\tilde{H}_t(h_t;s_{-i,t})}U^A(\mu_i; \tilde{h}_t) \\ &= E_{\tilde{H}_t^A(\mu_i;h_t;s_{-i,t})}U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}_t^W(\mu_i;h_t;s_{-i,t})}U^A(\mu_i; \tilde{h}_t) \end{aligned}$$

$$\begin{aligned} U^W(\mu_i; h_t) &= E_{\tilde{H}_t(h_t;s_{-i,t})}U^W(\mu_i; \tilde{h}_t) \\ &= E_{\tilde{H}_t^A(\mu_i;h_t;s_{-i,t})}U^W(\mu_i; \tilde{h}_t) + E_{\tilde{H}_t^W(\mu_i;h_t;s_{-i,t})}U^W(\mu_i; \tilde{h}_t) \end{aligned}$$

Since

$$\forall \tilde{h}_t \in \tilde{H}_t^A(\mu_i; h_t; s_{-i,t}), U^A(\mu_i; \tilde{h}_t) \geq U^W(\mu_i; \tilde{h}_t)$$

$$\forall \tilde{h}_t \in \tilde{H}_t^W(\mu_i; h_t; s_{-i,t}), U^W(\mu_i; \tilde{h}_t) \geq U^A(\mu_i; \tilde{h}_t)$$

we have

$$E_{\tilde{H}_t^A(\mu_i;h_t;s_{-i,t})}U^A(\mu_i; \tilde{h}_t) + E_{\tilde{H}_t^W(\mu_i;h_t;s_{-i,t})}U^W(\mu_i; \tilde{h}_t) \geq max\{U^A(\mu_i; h_t); U^W(\mu_i; h_t; s_{-i,t})\}$$

All the remaining individuals prefer to wait for the new information and make the appropriate decision. Thus, they welcome the new information in the ex ante sense.

The announcements of future disclosure of public information will increase the individuals' incentive to wait until its disclosure. In this case, some individuals may be worse off. For example, for the continuous private signal space, given the original equilibrium strategy profile $\{\mu_t^*(h_t)\}_t$, now at the beginning of period τ there is an announcement saying that in period $\tau + T$ there will be a disclosure of public information. With this announcement the equilibrium strategy profile changes to $\{\tilde{\mu}_t^*(h_t)\}_t$. Then for any history h_t, we have $\tilde{\mu}_t^*(h_t) \geq \mu_t^*(h_t)$ $\forall \tau \leq t < \tau + T$. There exists the possibility that $\tilde{\mu}_{t+1}^*(h_{t+1}) < \mu_i \leq \mu_t^*(h_t)$. For these individuals, they will upgrade in period $t+1$ in both equilibria. But with less information in the new equilibrium, they are worse off.

(ii) Disclosure of public information in the waiting herding phase

Similar to the disclosure of public information in the strategic phase, if there is a disclosure of public information in the waiting herding phase, either unexpected or expected, all the remaining individuals welcome the new information. They prefer to wait for the new information and make the appropriate decision. The waiting herding phase is indeed not robust as the disclosure of public information only needs to induce the most optimistic individuals among the remainders to upgrade. A small amount of positive information about the new software package A can shatter a waiting herding phase. Then a new strategic phase starts. Everyone is better off in the ex ante sense.

Since in the waiting herding phase everyone has already waited, the announcements of future disclosure of public information even more greatly increase the incentive to wait. The waiting herding continues until the disclosure of public information happens.

(iii) Multiple disclosures of public information

Unlike BHW, even though multiple disclosures of public information can eventually shatter a waiting herding phase, they cannot always rule out the possibility that individuals settle into the wrong upgrade herding. Suppose the multiple disclosures of public information eventually reveal that waiting is the better choice. But as long as the individuals are optimistic enough, they will not wait for the possible future multiple disclosures of public information. Furthermore, the upgrade herding phase could start before the true value of the new software package A is revealed. The game may end with the wrong upgrade herd conclusion. ■

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CHAPTER 2

Robust Herding with Endogenous Ordering and Two-Sided Commitment

2.1 Introduction

How do people make sequential decisions under imperfect information? One may learn from his own experiences or from other people's choices. For instance, individuals currently using a particular software package may also have the choice of upgrading to a new software package. They may have some knowledge about the new software package. But if the new software package is brand new and private information is limited, individuals may be inclined to wait for other people to discourse more information about the newly released software before they take any action. If the information previously aggregated dominates their own private information, individuals ignore their own private information and follow their predecessors – **herding** occurs.[1] Herding prevents the aggregation of information. Therefore, the initial realization of signals can have long-term consequences and herd behavior is often error prone. The decisions of the first few individuals' can have a disproportional effect.

[1]Çelen and Kariv (2004) attempt to make the distinction between herding and information cascades. They point out that in a herd, individuals choose the same action; but they may have acted differently if the realization of private signals had been different. In an information cascade, individuals ignore their own private information and follow their predecessors. Thus, information cascades in Çelen and Kariv (2004) are equivalent to herding in this paper.

Bikhchandani, Hirshleifer, and Welch (1992), hereafter BHW, and Banerjee (1992) investigate herd behavior under **exogenous ordering**, in which the decision ordering is exogenously given and only one individual moves in each period. The restaurant example in Banerjee (1992) may fit the exogenous ordering setting.[2] But in many other cases, **endogenous ordering** which allows individuals to choose the time of acting or waiting may be more appropriate. For instance, when individuals decide to buy a new car or computer, they have the option to buy immediately or to wait. With endogenous ordering, there exist strategic interactions among decision makers. Due to the free-rider problem, some decision makers may have incentives to delay their decisions and learn from other decision makers, while others make decisions immediately if they feel confident that their decisions will produce desirable results. Furthermore, more than one individual can act or wait during the same period and consequently their decisions can be clustered together. Thus, under the endogenous ordering setting, the insight will be completely different from that under the exogenous ordering setting. Our main question of inquiry is: if we allow decision makers to choose the time of acting or waiting, will herd behavior be more or less error prone?

Continuing with the software upgrading example, there is a new software package A available for upgrading. Individuals are currently using a software package B. It is known that with some prior probability A is better than B. Each individual also gets a private signal indicating whether A is better or not. Upgrading to A is an irreversible choice. Once they upgrade to A, they are committed to their decisions.[3] But there is no commitment to continuing using

[2]In the restaurant example in Banerjee (1992), there are two restaurants next to each other. Individuals arrive at the restaurants in sequence. Observing the choices made by people before them, they decide on either one of the two restaurants.

[3]There exists extremely high "disruption costs" involved in upgrading. In other words, we could see this upgrade as a perpetual American call option. Individuals are free to exercise the option at any time they want. But once they exercise the option, they cannot reverse their

B. If individuals have not upgraded, they continue to have the option of doing so. Thus, the software upgrading example belongs to the setting of **one-sided commitment**.

In contrast, the restaurant example in Banerjee (1992) is a **two-sided commitment** decision problem. Individuals choose between two restaurants. Choosing either one of the two restaurants is irreversible. Once an individual chooses one restaurant, he cannot go to the other any more. For exogenous ordering, one-sided commitment is equivalent to two-sided commitment because once an individual chooses A or B at his turn, he is out of the game and cannot change his decision any more. But for endogenous ordering, individuals in a one-sided commitment decision problem have two choices: A or B. If they choose A, they cannot change. If they choose B, they still have the option of choosing A later. Individuals in a two-sided commitment decision problem have three choices: A, B or wait. If they choose A or B, they cannot change. If they choose to wait, they still have the option of choosing A or B later. In other words, waiting is equivalent to choosing B in a one-sided commitment decision problem with endogenous ordering.

In this paper we concentrate on the two-sided commitment case.[4] We analyze an endogenous ordering sequential decision model in which decision makers are allowed to choose the time of acting (exercising a risky investment option A or a safe investment option B) or waiting. Compared with the one-sided commitment case, decision makers now have the third choice, exercising a safe investment option B. To emphasize the information aspect, we focus on pure information externalities: each decision maker's payoff only depends on his own action and the state of nature.

decision.

[4]Its companion (Zhang 2007a) investigates the one-sided commitment case.

Results obtained in the two-sided commitment case are similar to those found in the one-sided commitment case. That is, with endogenous ordering, if the number of decision makers is large and decision makers are patient enough, at any fixed time, nearly all decision makers wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information. However, one striking result is that with endogenous ordering and two-sided commitment, even though waiting forever is a dominated strategy, if the number of decision makers is large and decision makers are patient enough, decision makers wait too long.

There are some papers which investigate the decision problem with endogenous ordering. For example, Chamley and Gale (1994) investigate a discrete time investment model which assumes the timing of decisions is endogenous, that is, individuals try to find the best place in the decision-making queue. In their model, there are only two types of individuals: those with investment options and those without. Those individuals without investment options are assumed to be passive. In contrast, in our model we allow for a finite or an infinite number of types of individuals. Given one's own signals, each individual decides whether to act (exercising a risky investment option A or a safe investment option B) immediately or to wait and learn the true value of the risky investment option A by observing other individuals' actions.

The rest of the paper is organized as follows. Section 2.2 provides the setup of a general model and shows the existence of a symmetric equilibrium with the monotonicity property. Then we characterize herd behavior under exogenous ordering and endogenous ordering and discuss our main results. Section 2.3 concludes.

2.2 The General Model

In this section, we first provide the basic setup of our general model. Then we characterize herd behavior under exogenous ordering and endogenous ordering.

2.2.1 Basic Setup

There are N individuals. All are rational and risk neutral. There are two investment options: a risky investment option A and a safe investment option B. Assume that the true value of A, denoted by V^A, is chosen by nature at the beginning of the game, and is unknown to the individuals. Individuals only know V^A follows some prior distribution $F_0(V^A)$, with density $f_0(V^A)$. The value of B, denoted by V^B, is known to the individuals. To emphasize the information aspect, we concentrate on pure information externalities: each individual's payoff only depends on his own action and the state of nature.

We focus on the case that both exercising A and exercising B are irreversible binary choices.[5] The indivisibility of the action space is important. As in Banerjee (1992), since the choices made by individuals are not sufficient statistics for the information they have, the error prone herding can occur.[6]

At the beginning of the game, individual i in the market freely observes some conditionally independent private signal $\mu_i \in [\underline{\mu}, \overline{\mu}]$, which follows some distribution $F(\mu_i|V^A)$, with density $f(\mu_i|V^A)$. Assume individuals are more likely to

[5] There exists extremely high "disruption costs" involved in upgrading. In other words, we could see these exercising as a perpetual American call option as in Grenadier (1999). In Grenadier (1999), decisions are made in continuous time and there is a state variable, which follows some exogenous continuous time stochastic process. In this paper, we assume discrete time decision and no exogenous state variable.

[6] Banerjee (1992) assumes a continuous action space and gets similar herding results as BHW. This is due to the degenerate payoff function as pointed out by BHW. Park (2001) assumes perfect observability. Therefore, in his model players share the same information and hidden information is not an issue.

get a higher private signal (indicating higher value of A) if the underlying V^A is higher.

Assumption 1: $F(\mu_i|V^A)$ satisfies the *Monotone Likelihood Ratio Property* (MLRP) with respect to V^A, i.e.

$$\frac{f(\mu_i|V_1^A)}{f(\mu_i|V_2^A)} \quad \text{increasing in} \quad \mu_i \quad \forall V_1^A > V_2^A$$

If individual i exercises A, B, or waits in period t, then in the following periods, everyone knows individual i's action in period t. The public information available at the beginning of period t is denoted by h_t, which includes the prior information of V^A, actions and the equilibrium strategy profile of all individuals before t. If an individual does not exercise both A and B, he gets reservation utility V^0, normalized to zero. The common discount factor is δ. Here, we assume $V^B > V^0 = 0$.[7]

2.2.2 Herd Behavior with Exogenous Ordering

If we assume the ordering of individuals is exogenous, in which only one individual moves in each period in an exogenously given order, then there are no strategic interactions among individuals. When it is one's turn to make a decision, he decides to exercise A or B given the current public information and his own private signal.[8]

The equilibrium decision rule is a sequence of critical values

$$\{\mu_t^*(h_t)\}_t$$

[7]Otherwise, if $V^B \leq V^0 = 0$, no one will choose B. We are back to the situation of one-sided commitment. See the companion paper Zhang (2007a) for details.

[8]Since $V^B > V^0 = 0$, no one will choose waiting given their decisions are once and for all. Therefore, for exogenous ordering, two-sided commitment is equivalent to one-sided commitment (Zhang 2007a).

such that the individual making the decision in period t exercises A if his private signal $\mu_t > \mu_t^*(h_t)$; otherwise, he exercises B.[9] We can see this sequence of critical values is not monotone. If the individual in period t exercises A, which indicates $\mu_t > \mu_t^*(h_t)$, this is "good" news for the individual in period $t+1$. Thus, in period $t+1$, $\mu_{t+1}^*(h_{t+1}) \leq \mu_t^*(h_t)$. Conversely, if the individual in period t exercises B, $\mu_{t+1}^*(h_{t+1}) \geq \mu_t^*(h_t)$.

The game is in the strategic phase when the sequence of critical values $\{\mu_t^*(h_t)\}_t$ fluctuates in between $\underline{\mu}$ and $\overline{\mu}$. In the strategic phase, each individual's decision depends on both the current public information and his own private signal.

Once the sequence of critical values $\{\mu_t^*(h_t)\}_t$ "breaks" either one of the boundaries, herding occurs. A herding phase starts in period τ if $\mu_\tau^*(h_\tau) = \underline{\mu}$. The individual in period τ will exercise A regardless of his own private signal. His decision is, therefore, uninformative to others. Thus, $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \underline{\mu}$ $\forall t > \tau$ (see figure 2.1). All the following individuals will exercise A.

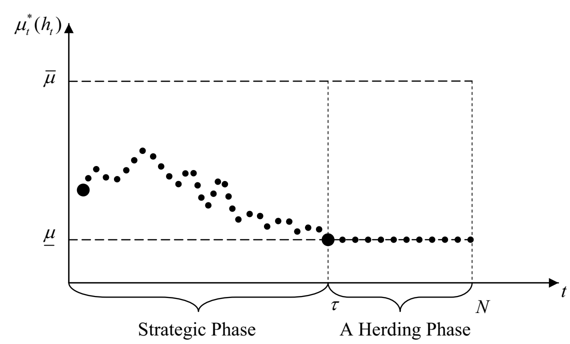

Figure 2.1: A herding with exogenous ordering

Similarly, B herding phase starts in period τ if $\mu_\tau^*(h_\tau) = \overline{\mu}$. All the following

[9]For notation simplicity, we assume the following tie-breaking rule: if an individual i is indifferent between exercising A and B, he exercises B whenever $\mu_i \in (\underline{\mu}, \overline{\mu}]$ and exercises A whenever $\mu_i = \underline{\mu}$.

individuals will exercise B and $\mu_t^*(h_t) = \mu_{t-1}^*(h_{t-1}) = \overline{\mu}$ $\forall t > \tau$ (see figure 2.2).

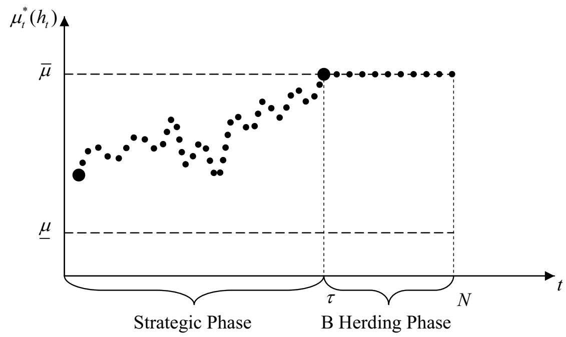

Figure 2.2: B herding with exogenous ordering

Since public information disclosed only needs to offset the information from the last individual's action before the herding phase starts, both A herding and B herding are not robust to the public disclosure of information. If in a certain period $N \geq t \geq \tau$ there is some public information disclosed such that $\underline{\mu} < \mu_t^*(h_t) < \overline{\mu}$, then the strategic phase starts again.

2.2.3 Herd Behavior with Endogenous Ordering

If we allow individuals to choose the time of acting (exercising A or B) or waiting, there exist strategic interactions among individuals.

The timing of endogenous ordering is as follows:

> In period 1, each individual decides whether or not to exercise A or B. If he does not exercise A or B in period 1, he gets reservation utility $V^0 = 0$ and has the option of exercising A or B later.
>
> In period 2, all the remaining individuals decide to exercise A, B or to wait after observing others' actions in period 1.

The subsequent periods are the same as period 2. The game continues until everyone exercises A or B. The time period is denoted by t, $t = 1, 2, 3, ...$

The benefit from waiting is the information revealed about A by other individuals. The cost of waiting is the difference between the gain from A or B and the reservation utility.

We first investigate the relationship between the incentive to wait and private information. We prove any possible symmetric equilibrium must be monotone with respect to personal private signals. Then, we show the existence and describe characteristics of a symmetric equilibrium with the monotonicity property by backward induction in two cases: a continuous private signal space and a finite discrete private signal space.[10]

2.2.3.1 Information and Incentives

The following remark shows that if an individual gets a higher private signal, given the same history, he believes that V^A will be higher, i.e., the posterior distribution of V^A satisfies MLRP with respect to private signals.

Remark 2.1

$$\frac{f(V^A|\mu_i, h_t)}{f(V^A|\mu_i', h_t)} \quad \textit{increasing in} \quad V^A \quad \forall \mu_i > \mu_i'$$

Proof. See the Appendix in the companion paper (Zhang 2007a). ■

The benefit from exercising A in period t for individual i is:

$$U^A(\mu_i; h_t) = E_{V^A|\mu_i;h_t} V^A \tag{2.1}$$

[10]Since the information disclosed through the backward induction construction process may not be monotone, the equilibrium is not necessarily unique.

The benefit from exercising B is known and constant in any period:

$$U^B = V^B$$

The benefit from waiting in period t for individual i is:

$$U^W(\mu_i; h_t; s_{-i,t}) = \delta E_{H_{t+1}(\mu_i;h_t;s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1}); U^B\}] \tag{2.2}$$

where $s_{-i,t}$ represents the strategy profile of all other individuals except for individual i starting from period t; $H_{t+1}(\mu_i; h_t; s_{-i,t})$ represents the set of histories at the beginning of period $t+1$ given μ_i, h_t and $s_{-i,t}$. From the above equation, we can solve the benefit from waiting forever $\underline{U}^W = 0$. Consequently, we have the following lemma.

Lemma 2.1 *Since the benefit from waiting forever* $\underline{U}^W = 0$ *and* $V^B > 0$*, individuals will never choose waiting forever. They will choose to either exercise* A *or* B *in a finite number of periods. Since both exercising* A *and* B *are irreversible binary choices, that is to say, the game ends in a finite number of periods.*

In this paper, we focus on the symmetric equilibrium. The next proposition proves that for any possible symmetric equilibrium, it must be monotone with respect to personal private signals. That is, individuals with private signals indicating higher value of A have a higher incentive to exercise A than to wait or to exercise B; individuals with private signals indicating higher value of A have a higher incentive to wait than to exercise B.

Proposition 2.1

(i) $U^A(\mu_i; h_t) - U^B$ *increasing in* $\mu_i \quad \forall h_t$

(ii) $U^W(\mu_i; h_t; s_{-i,t}) - U^B$ *increasing in* $\mu_i \quad \forall h_t; s_{-i,t}$

(iii) $U^A(\mu_i; h_t) - U^W(\mu_i; h_t; s_{-i,t})$ *increasing in* $\mu_i \quad \forall h_t; s_{-i,t}$

Proof. See the Appendix. ■

2.2.3.2 Symmetric Equilibrium with the Monotonicity Property

Proposition 2.2 *There exists a symmetric equilibrium with the following monotonicity property.*

(i) Case I: Continuous private signal space

The equilibrium strategy profile is a sequence of decreasing critical values $\{\mu_t^A(h_t)\}_t$ *and a sequence of increasing critical values* $\{\mu_t^B(h_t)\}_t$*. In period* t *with history* h_t*, individuals with* $\mu > \mu_t^A(h_t)$ *exercise A; individuals with* $\mu < \mu_t^B(h_t)$ *exercise B;others wait.*

Case II: Finite discrete private signal space

The equilibrium strategy profile is a sequence of decreasing critical values $\{\mu_t^A(h_t)\}_t$ *and a sequence of increasing critical values* $\{\mu_t^B(h_t)\}_t$*, with two sequences of probability* $\{p_{\mu_t^A(h_t)}\}_t$ *and* $\{p_{\mu_t^B(h_t)}\}_t$*. In period* t *with history* h_t*, individuals with* $\mu > \mu_t^A(h_t)$ *exercise A; individuals with* $\mu < \mu_t^B(h_t)$ *exercise B; the critical type individuals with* $\mu = \mu_t^A(h_t)$ *exercise A with probability* $p_{\mu_t^A(h_t)}$*; the critical type individuals with* $\mu = \mu_t^B(h_t)$ *exercise B with probability* $p_{\mu_t^B(h_t)}$*; others wait.*[11]

(ii) Large Number and Patient Individuals: *If number of individuals is large and individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed.*

Proof. See the Appendix. ■

[11] *For simplicity,* $p_{\mu_t^A(h_t)} < 1$ *and* $p_{\mu_t^B(h_t)} < 1$ *are chosen in the construction process so that there is the possibility for the* $\mu_t^A(h_t)$ *and* $\mu_t^B(h_t)$ *types of individuals to remain in the game in period* $t+1$*. Moreover,* $\mu_t^A(h_t)$ *is the highest type in period* $t+1$*;* $\mu_t^B(h_t)$ *is the lowest type in period* $t+1$*.*

Part (i) is from the construction in the above proposition. For part (ii), if the number of individuals is large and $\mu_1^A(h_1)$ strictly smaller than $\overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}^A}$ strictly greater than 0), by the Law of Large Numbers, the true value of the risky investment option A will be (approximately) revealed in the second period. Similarly, if the number of individuals is large and $\mu_1^B(h_1)$ strictly greater than $\underline{\mu}$ (finite discrete private signal space: $p_{\underline{\mu}^B}$ strictly greater than 0), by the Law of Large Numbers, the true value of the risky investment option A will be (approximately) revealed in the second period. In this case, if individuals are patient enough, then all individuals will wait in period 1 such that $\mu_1^A(h_1) = \overline{\mu}$ and $\mu_1^B(h_1) = \underline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}^A} = 0$ and $p_{\underline{\mu}^B} = 0$). This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game: either $\mu_t^A(h_t) = \mu_{t-1}^A(h_{t-1})$ or $\mu_t^A(h_t) \approx \mu_{t-1}^A(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}^A} = 0$ or $p_{\overline{\mu}^A} \approx 0$); and either $\mu_t^B(h_t) = \mu_{t-1}^B(h_{t-1})$ or $\mu_t^B(h_t) \approx \mu_{t-1}^B(h_{t-1})$ (finite discrete private signal space: either $p_{\underline{\mu}^B} = 0$ or $p_{\underline{\mu}^B} \approx 0$). Thus, at any fixed time, there is a negligible proportion of individuals exercising A or B and so is the information disclosed.

From the above proposition, with endogenous ordering the sequences of critical values are monotone. Intuitively, in any period t, all the individuals with $\mu > \mu_t^A(h_{t-1})$ exercise A and all the individuals with $\mu < \mu_t^B(h_{t-1})$ exercise B before t. In period t, we only need to consider the individuals with private signals between $\mu_t^B(h_{t-1})$ and $\mu_{t-1}^A(h_{t-1})$. Thus, $\mu_t^A(h_t) \leq \mu_{t-1}^A(h_{t-1})$ and $\mu_t^B(h_t) \geq \mu_{t-1}^B(h_{t-1})$.

A herding occurs in period τ when $\mu_\tau^A(h_\tau) = \mu_{\tau-1}^B(h_{\tau-1})$ (finite discrete signal space: $\mu_\tau^A(h_\tau) = \mu_{\tau-1}^B(h_{\tau-1}); p_{\mu_\tau^A(h_\tau)} = 1$). All the remaining individuals exercise A in period τ regardless of their own private signal, and then the game ends (see figure 2.3). B herding occurs in period τ when $\mu_{\tau}^{B}(h_{\tau}) = \mu_{\tau-1}^{A}(h_{\tau-1})$ (finite

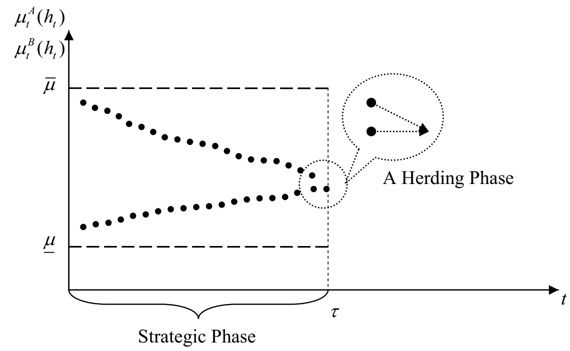

Figure 2.3: A herding with endogenous ordering

discrete signal space: $\mu_{\tau}^{B}(h_{\tau}) = \mu_{\tau-1}^{A}(h_{\tau-1}); p_{\mu_{\tau}^{B}(h_{\tau})} = 1$). All the remaining individuals exercise B in period τ regardless of their own private signal, and then the game ends (see figure 2.4).

If the game falls into either A herding phase or B herding phase, disclosure of public information after τ will not have any effect since A herding phase or B herding phase only lasts one period. Therefore, both A herding phase and B herding phase are robust to the future public disclosure of information.

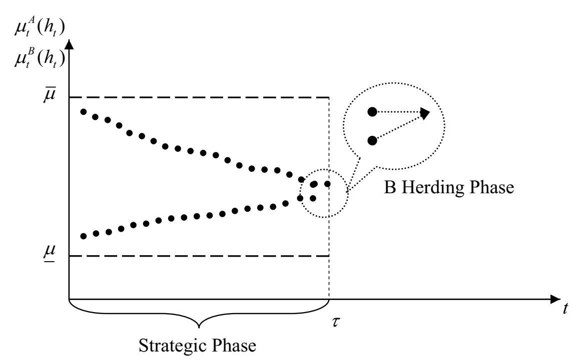

Figure 2.4: B herding with endogenous ordering

2.2.4 Robustness

We summarize the results of the impact of public information disclosure on herding behavior under exogenous and endogenous ordering settings respectively in the following table.[12]

Table 2.1: Robustness with respect to Disclosure of Public Information

		exogenous ordering		endogenous ordering	
		A herding	B herding	A herding	B herding
time of disclosure	$N \geq t > \tau$	Not Robust	Not Robust	Robust	Robust
of public information	$t > N$	Robust	Robust	Robust	Robust

By the game construction, under the exogenous ordering setting, the game lasts exactly N periods. Disclosure of public information after period N will not have any effect. Under the endogenous ordering setting, both A herding phase and B herding phase only last one period. Disclosure of public information after τ will not have any effect.

2.2.5 Expected Number of Correct Choices

Proposition 2.3 *In the general model, if the number of individuals is large and individuals are patient enough, exogenous ordering is more efficient than endogenous ordering in terms of inducing a larger expected number of correct choices, if individuals can be forced to move with an exogenous order.*

Proof. From proposition 2.2, with endogenous ordering, if the number of in-

[12]Here, we only talk about the unexpected disclosure of public information. The companion paper Zhang (2007a) investigates more variations of public information disclosure.

dividuals is large and individuals are patient enough, at any fixed time, nearly all individuals wait due to the negligible information disclosed. In contrast, in the self-decision case, each individual still utilizes his own private information and the prior probabilities. Exogenous ordering is even better since it tends to aggregate more information by forcing some of the individuals to make decisions in some given periods.If individuals are patient enough, as long as the private signals and the prior probabilities are informative, exogenous ordering is more likely to induce a larger expected number of correct choices than endogenous ordering. ■

2.3 Conclusion

In this paper, we investigate herd behavior of sequential decisions under imperfect information with two-sided commitment. We provide a framework of endogenous ordering to allow decision makers to choose the time of acting (exercising a risky investment option A or a safe investment option B) or waiting. We show the existence and characteristics of the equilibrium. We find that herd behavior under endogenous ordering is not necessarily less error prone than herd behavior under exogenous ordering due to the free-rider problem. In particular, if the number of individuals is large and individuals are patient enough, under endogenous ordering nearly all individuals are willing to wait and free-ride on others. Consequently, nearly all individuals wait due to the negligible information disclosed. In this case, if decision makers can be forced to move with an exogenous order, the resulting equilibrium is more efficient because exogenous ordering tends to aggregate more information. That is to say, more "freedom" may not be better.

Another feature of the endogenous ordering framework is that in a herding phase, all the remaining individuals either act (exercising a risky investment option A or a safe investment option B) immediately regardless of their own private signals. Thus, there exists an investment surge (either A or B) when herding starts. Compared with the exogenous ordering setting, disclosure of public information has a completely different impact on the strategic and herding behavior of individuals. In particular, if the game is in A herding phase or B herding phase, all the remaining individuals exercising either A or B immediately and the game ends in one period. Further disclosure of public information will not have any effect.

2.4 Appendix

Proof of Proposition 2.1

(i) By Remark 1, $f(V^A|\mu, h_t)$ satisfies MLRP with respect to μ. According to Landsberger and Meilijson (1990), $F(V^A|\mu_i, h_t)$ first order stochastic dominates (FOSD) $F(V^A|\mu_i', h_t)$ for any $\mu_i > \mu_i'$. So, $U^A(\mu_i; h_t) \geq U^A(\mu_i'; h_t)$ for any h_t and $\mu_i > \mu_i'$. Since U^B is a constant, we have $U^A(\mu_i; h_t) - U^B$ increasing in μ_i, $\forall h_t$.

(ii) Let us check the benefit of waiting for individual i who has a private signal μ_i. The set of histories $H_{t+1}(\mu_i; h_t; s_{-i,t})$ can be decomposed into three disjoint sets: $H_{t+1}^A(\mu_i; h_t; s_{-i,t})$, $H_{t+1}^W(\mu_i; h_t; s_{-i,t})$ and $H_{t+1}^B(\mu_i; h_t; s_{-i,t})$, where $H_{t+1}^A(\mu_i; h_t; s_{-i,t})$ is the set of histories in period $t+1$ in which individual i will exercise A according to some strategy s_i of individual i; $H_{t+1}^W(\mu_i; h_t; s_{-i,t})$ is the set of histories in period $t+1$ in which individual i will wait according to some strategy s_i of individual i; and $H_{t+1}^B(\mu_i; h_t; s_{-i,t})$ is the set of histories in period $t+1$ in which individual i will exercise B according to some strategy s_i of individual i.

$$U^W(\mu_i; h_t; s_{-i,t}) = \delta E_{H_{t+1}(\mu_i;h_t;s_{-i,t})}[max\{U^A(\mu_i; h_{t+1}); U^W(\mu_i; h_{t+1}; s_{-i,t+1})\}; U^B]$$
$$= \delta[E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})}U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) + E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})}U^B] \tag{2.3}$$

Similarly, the benefit of waiting for individual i who has a lower private signal $\mu'_i < \mu_i$ is as following

$$U^W(\mu'_i; h_t; s_{-i,t}) = \delta E_{H_{t+1}(\mu'_i;h_t;s_{-i,t})}[max\{U^A(\mu'_i; h_{t+1}); U^W(\mu'_i; h_{t+1}; s_{-i,t+1})\}; U^B]$$
$$= \delta[E_{H^A_{t+1}(\mu'_i;h_t;s_{-i,t})}U^A(\mu'_i; h_{t+1}) + E_{H^W_{t+1}(\mu'_i;h_t;s_{-i,t})}U^W(\mu'_i; h_{t+1}; s_{-i,t+1}) + E_{H^B_{t+1}(\mu'_i;h_t;s_{-i,t})}U^B] \tag{2.4}$$

By Lemma 2.1, the game ends in a finite number of periods. Suppose the game ends in period T. No more new information is disclosed after period T, which means $U^W(\mu_i; h_t; s_{-i,t}) = U^W(\mu'_i; h_t; s_{-i,t}) = 0 \ \forall t > T$.

Using mathematical induction, suppose $U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \geq U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$ for some t. We need to prove $U^W(\mu_i; h_t; s_{-i,t}) \geq U^W(\mu'_i; h_t; s_{-i,t})$. By equation 2.3 and 2.4,

$$\begin{aligned}
U^W(\mu_i; h_t; s_{-i,t}) = \delta[&E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu'_i;h_t;s_{-i,t})}U^A(\mu_i; h_{t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu'_i;h_t;s_{-i,t})}U^A(\mu_i; h_{t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu'_i;h_t;s_{-i,t})}U^A(\mu_i; h_{t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu'_i;h_t;s_{-i,t})}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu'_i;h_t;s_{-i,t})}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu'_i;h_t;s_{-i,t})}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu'_i;h_t;s_{-i,t})}U^B \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu'_i;h_t;s_{-i,t})}U^B \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu'_i;h_t;s_{-i,t})}U^B]
\end{aligned}$$

$$
\begin{aligned}
U^W(\mu_i';h_t;s_{-i,t}) = \delta[&E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})}U^A(\mu_i';h_{t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})}U^W(\mu_i';h_{t+1};s_{-i,t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})}U^B \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})}U^A(\mu_i';h_{t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})}U^W(\mu_i';h_{t+1};s_{-i,t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})}U^B \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})}U^A(\mu_i';h_{t+1}) \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})}U^W(\mu_i';h_{t+1};s_{-i,t+1}) \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})\cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})}U^B]
\end{aligned}
$$

Let us discuss the cases of joint history sets one by one.

For $h_{t+1} \in [H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^A(\mu_i;h_{t+1}) \geq U^A(\mu_i';h_{t+1})$$

For $h_{t+1} \in [H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^A(\mu_i;h_{t+1}) \geq U^W(\mu_i;h_{t+1};s_{-i,t+1}) \geq U^W(\mu_i';h_{t+1};s_{-i,t+1})$$

For $h_{t+1} \in [H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^A(\mu_i;h_{t+1}) \geq U^B$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^W(\mu_i;h_{t+1}) \geq U^A(\mu_i;h_{t+1}) \geq U^A(\mu_i';h_{t+1})$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^W(\mu_i;h_{t+1};s_{-i,t+1}) \geq U^W(\mu_i';h_{t+1};s_{-i,t+1})$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})]$,

$$U^W(\mu_i;h_{t+1}) \geq U^B$$

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since

$$U^B \geq U^A(\mu_i; h_{t+1}) \geq U^A(\mu'_i; h_{t+1})$$

$H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu'_i; h_t; s_{-i,t})$ is an empty set.

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since

$$U^B \geq U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \geq U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$$

$H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})$ is an empty set.

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^B_{t+1}(\mu'_i; h_t; s_{-i,t})]$,

$$U^B \geq U^B$$

Thus, for all joint history sets, the arguments from $U^W(\mu_i; h_t; s_{-i,t})$ is greater than or equal to those from $U^W(\mu'_i; h_t; s_{-i,t})$. Thus, $U^W(\mu_i; h_t; s_{-i,t}) \geq U^W(\mu'_i; h_t; s_{-i,t})$ for any t. Since U^B is a constant, we have

$$U^W(\mu_i; h_t; s_{-i,t}) - U^B \quad \text{increasing in} \quad \mu_i \quad \forall h_t; s_{-i,t}$$

(iii) By the Martingale property,

$$\begin{aligned} U^A(\mu_i; h_t) = {} & E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) \\ & + E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) \end{aligned}$$

By equation 2.3 and 2.4, for any non-negative integer j

$$\begin{aligned} U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) = {} & (1 - \delta^{j+1}) E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) \\ & + E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \\ & + E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^B] \end{aligned} \quad (2.5)$$

$$\begin{aligned} U^A(\mu'_i; h_t) - \delta^j U^W(\mu'_i; h_t; s_{-i,t}) = {} & (1 - \delta^{j+1}) E_{H^A_{t+1}(\mu'_i;h_t;s_{-i,t})} U^A(\mu'_i; h_{t+1}) \\ & + E_{H^W_{t+1}(\mu'_i;h_t;s_{-i,t})} [U^A(\mu'_i; h_{t+1}) - \delta^{j+1} U^W(\mu'_i; h_{t+1}; s_{-i,t+1})] \\ & + E_{H^B_{t+1}(\mu'_i;h_t;s_{-i,t})} [U^A(\mu'_i; h_{t+1}) - \delta^{j+1} U^B] \end{aligned} \quad (2.6)$$

By Lemma 2.1, the game ends in a finite number of periods. Suppose the game ends in period T. No more new information is disclosed after period T, which means $U^W(\mu_i; h_t; s_{-i,t}) = U^W(\mu_i'; h_t; s_{-i,t}) = 0 \ \forall t > T$. Thus, $U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) \geq U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t}) \ \forall t > T$.

Using mathematical induction, suppose $U^A(\mu_i; h_{t+1}) - \delta^j U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \geq U^A(\mu_i'; h_{t+1}) - \delta^j U^W(\mu_i'; h_{t+1}; s_{-i,t+1})$ for some t. We need to prove $U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) \geq U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t})$. By equation 2.5 and 2.6,

$$
\begin{aligned}
U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) &= (1-\delta^{j+1})[E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i; h_{t+1}) + E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i; h_{t+1})] \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^W(\mu_i; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^B] \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^B] \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^B]
\end{aligned}
$$

$$
\begin{aligned}
U^A(\mu_i'; h_t) - \delta^j U^W(\mu_i'; h_t; s_{-i,t}) &= (1-\delta^{j+1}) E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i'; h_{t+1}) \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^W(\mu_i'; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^A_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^B] \\
&+ (1-\delta^{j+1}) E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i'; h_{t+1}) \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^W(\mu_i'; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^W_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i; h_{t+1}) - \delta^{j+1} U^B] \\
&+ (1-\delta^{j+1}) E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^A_{t+1}(\mu_i';h_t;s_{-i,t})} U^A(\mu_i'; h_{t+1}) \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^W_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^W(\mu_i'; h_{t+1}; s_{-i,t+1})] \\
&+ E_{H^B_{t+1}(\mu_i;h_t;s_{-i,t}) \cap H^B_{t+1}(\mu_i';h_t;s_{-i,t})} [U^A(\mu_i'; h_{t+1}) - \delta^{j+1} U^B]
\end{aligned}
$$

Let us discuss the cases of joint history sets one by one.

For $h_{t+1} \in [H^A_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu_i'; h_t; s_{-i,t})]$,

$$(1-\delta^{j+1}) U^A(\mu_i; h_{t+1}) \geq (1-\delta^{j+1}) U^A(\mu_i'; h_{t+1})$$

For $h_{t+1} \in [H^A_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since $U^W(\mu'_i; h_{t+1}; s_{-i,t+1}) \geq U^A(\mu'_i; h_{t+1})$,

$$(1-\delta^{j+1})U^A(\mu_i; h_{t+1}) \geq (1-\delta^{j+1})U^A(\mu'_i; h_{t+1}) \geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$$

For $h_{t+1} \in [H^A_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^B_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since $U^B \geq U^A(\mu'_i; h_{t+1})$,

$$(1 - \delta^{j+1})U^A(\mu_i; h_{t+1}) \geq (1 - \delta^{j+1})U^A(\mu'_i; h_{t+1}) \geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^B$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since $U^A(\mu'_i; h_{t+1}) \geq U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$,

$$\begin{aligned} U^A(\mu_i; h_{t+1}) - \delta^{j+1}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) &\geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^W(\mu'_i; h_{t+1}; s_{-i,t+1}) \\ &\geq (1 - \delta^{j+1})U^A(\mu'_i; h_{t+1}) \end{aligned}$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})]$,

$$U^A(\mu_i; h_{t+1}) - \delta^{j+1}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$$

For $h_{t+1} \in [H^W_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^B_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since $U^B \geq U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$,

$$\begin{aligned} U^A(\mu_i; h_{t+1}) - \delta^{j+1}U^W(\mu_i; h_{t+1}; s_{-i,t+1}) &\geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^W(\mu'_i; h_{t+1}; s_{-i,t+1}) \\ &\geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^B \end{aligned}$$

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since

$$U^B \geq U^A(\mu_i; h_{t+1}) \geq U^A(\mu'_i; h_{t+1})$$

$H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^A_{t+1}(\mu'_i; h_t; s_{-i,t})$ is an empty set.

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})]$, since

$$U^B \geq U^W(\mu_i; h_{t+1}; s_{-i,t+1}) \geq U^W(\mu'_i; h_{t+1}; s_{-i,t+1})$$

$H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^W_{t+1}(\mu'_i; h_t; s_{-i,t})$ is an empty set.

For $h_{t+1} \in [H^B_{t+1}(\mu_i; h_t; s_{-i,t}) \cap H^B_{t+1}(\mu'_i; h_t; s_{-i,t})]$,

$$U^A(\mu_i; h_{t+1}) - \delta^{j+1}U^B \geq U^A(\mu'_i; h_{t+1}) - \delta^{j+1}U^B$$

Thus, for all joint history sets, the arguments from $U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t})$ is greater than or equal to those from $U^A(\mu'_i; h_t) - \delta^j U^W(\mu'_i; h_t; s_{-i,t})$. Thus, $U^A(\mu_i; h_t) - \delta^j U^W(\mu_i; h_t; s_{-i,t}) \geq U^A(\mu'_i; h_t) - \delta^j U^W(\mu'_i; h_t; s_{-i,t})$ for any t. Let $j = 0$, we have $U^A(\mu_i; h_t) - U^W(\mu_i; h_t; s_{-i,t})$ increasing in μ_i, $\forall h_t; s_{-i,t}$. ■

Proof of Proposition 2.2

(i) Case I: Continuous private signal space

Let $\mathcal{G}_n(h_t)$ represent the subgame starting from period t with history h_t, where n is the number of individuals remaining in this subgame.[13] Use backward induction.

Step 1 Start from the subgame with only one individual, $\mathcal{G}_1(h_t)$. We can find a critical value $\mu_t^*(h_t)$ which is the solution of $U^A(\mu_t^*(h_t); h_t) = U^B$. In this case, $\mu_t^*(h_t) = \mu_t^A(h_t) = \mu_t^B(h_t)$. If the individual's private signal $\mu > \mu_t^*(h_t)$, he will exercise A; otherwise, he exercises B.

Step 2 Now consider the subgame with two individuals $\mathcal{G}_2(h_t)$. By Lemma 2.1, this subgame ends in a finite number of periods, which means in some period, say T, $\mu_T^A(h_T) = \mu_T^B(h_T)$. This subgame could have been end or evolved to a subgame with only one individual in some period $t \leq s < T$, depending on the realization of private signals of these two individuals. Thus, to find the pair of critical values $\{\mu_t^A(h_t), \mu_t^B(h_t)\}$ in period t, we must construct the entire scheme of pairs of critical values from period t to T,$\{\mu_s^A(h_s), \mu_s^B(h_s)\}_{s=t}^T$, with $\mu_T^A(h_T) = \mu_T^B(h_T)$.

Using backward induction, start with any arbitrary $\mu_T^A(h_T) = \mu_T^B(h_T)$. Certainly, it must lie in between $\{\mu_{t-1}^A(h_{t-1}), \mu_{t-1}^B(h_{t-1})\}$. Since T is the last period, we have

$$U^A(\mu_T^A(h_T); h_T) = U^B \geq U^W(\mu_T^A(h_T); h_T; s_{-i,T})$$

h_T is the history till period T, including h_t, 2 individuals remaining in the game, and their private signals are in between $\{\mu_{T-1}^A(h_{T-1}), \mu_{T-1}^B(h_{t-1})\}$.

[13] n must be compatible with h_t.

$s_{-i,T}$ is the strategy profile of the opponent, in which the opponent will exercise A if his private signal is greater than $\mu_T^A(h_T)$, otherwise exercise B. We have one equation, one inequality, and only 2 unknowns $\mu_{T-1}^A(h_{T-1})$ and $\mu_{T-1}^B(T_{t-1})$. Thus, we can solve the range of $\{\mu_{T-1}^A(h_{T-1}), \mu_{T-1}^B(h_{t-1})\}$, in which the game ends in period T with $\mu_T^A(h_T) = \mu_T^B(h_T)$.

Then back to period $T-1$, we have the following equations.

$$U^A(\mu_{T-1}^A(h_{T-1}); h_{T-1}) = U^W(\mu_{T-1}^A(h_{T-1}); h_{T-1}; s_{-i,T-1})$$
$$U^B = U^W(\mu_{T-1}^B(h_{T-1}); h_{T-1}; s_{-i,T-1})$$

Similarly, we can solve the range of $\{\mu_{T-2}^A(h_{T-2}), \mu_{T-2}^B(h_{t-2})\}$, in which the game in period $T-1$ with the pair of critical values $\{\mu_{T-1}^A(h_{T-1}), \mu_{T-1}^B(h_{t-1})\}$.

Backward till $\{\mu_{t-1}^A(h_{t-1}), \mu_{t-1}^B(h_{t-1})\}$ is in the range of pair of critical values such that the game ends in period T with $\mu_T^A(h_T) = \mu_T^B(h_T)$. In this process, we might try different values of $\mu_T^A(h_T) = \mu_T^B(h_T)$.

⋮

⋮

Step N Continue to the subgame with N individuals $\mathcal{G}_N(h_t)$. Similarly as previous steps, start with any arbitrary $\mu_T^A(h_T) = \mu_T^B(h_T)$ to construct the entire scheme of of pairs of critical values from period t to T,$\{\mu_s^A(h_s), \mu_s^B(h_s)\}_{s=t}^T$, with $\mu_T^A(h_T) = \mu_T^B(h_T)$. Try different values of $\mu_T^A(h_T) = \mu_T^B(h_T)$ till $\{\mu_{t-1}^A(h_{t-1}), \mu_{t-1}^B(h_{t-1})\}$ is in the range of pair of critical values such that the game ends in period T with $\mu_T^A(h_T) = \mu_T^B(h_T)$.

We can see in the final Step N if we replace h_t with h_1 then $\mathcal{G}_N(h_t)$ is the original game.

Case II: Finite discrete private signal space

Denote the private signal space by $\{\mu_1, \mu_2, \ldots, \mu_K\}$, where $\mu_1 < \mu_2 < \ldots < \mu_K$. The strategy profile starting from period t, $s_t = \{P_\tau\}_{\tau=t}^{\infty}$, where $P_\tau = \{p_{\mu_k^A,\tau}, p_{\mu_k^B,\tau}\}_{k=1}^{K}$ and $p_{\mu_k^A,\tau}, p_{\mu_k^B,\tau}$ represents the probability of type μ_k exercising A and B in period τ. For μ_i, $U^A(\mu_i; h_t) - U^W(\mu_i; h_t; \{P_\tau\}_{\tau=t}^{\infty})$, $U^A(\mu_i; h_t) - U^B$, and $U^W(\mu_i; h_t; \{P_\tau\}_{\tau=t}^{\infty}) - U^B$ are continuous in $p_{\mu_k,\tau}$ $\forall \mu_k, \tau$. Let $\mathcal{G}_M(h_t)$ represent the subgame starting from period t with history h_t, where M is the set of possible types remaining in this subgame.[14]

By Proposition 2.1, with a finite number of individuals and a finite number of individual types, backward induction can be used to construct the symmetric equilibrium through the following steps.

Step 1 Start from the subgame $\mathcal{G}_{\{\mu_1\}}(h_t)$ with only one type of individual μ_1. That is, $M = \{\mu_1\}$. Since all the information is disclosed,

$$U^W(\mu_1; h_t; \{p_{\mu_1^A,t}, p_{\mu_1^B,t}\}) = \underline{U}^W = 0$$

There are three possible cases:

1.1 If $U^A(\mu_1; h_t) < U^B$, $\{p_{\mu_1^B,t} = 1\}$ is the equilibrium and game ends.

1.2 If $U^A(\mu_1; h_t) = U^B$, individuals are indifferent in between exercising A and B. By some tie-breaking rule, individuals will exercise A or B in period t and game ends.

1.3 If $U^A(\mu_1; h_t) > U^B$, $\{p_{\mu_1^A,t} = 1\}$ is the equilibrium and game ends.

Step 2 Now consider the subgame $\mathcal{G}_{\{\mu_1,\mu_2\}}(h_t)$ with two possible types of individuals μ_1, μ_2. Thus, $M = \{\mu_1, \mu_2\}$. There are three possible cases:

2.1 If $U^B \geq U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1^B,t} = p_{\mu_2^B,t} = 1, p_{\mu_1^A,t} = p_{\mu_2^A,t} = 0\}$ is an equilibrium strategy profile in period t.

[14] M must be compatible with h_t.

2.2 If $U^A(\mu_2; h_t) > U^B \geq U^A(\mu_1; h_t)$, then for the equilibrium strategy profile in period t, $p_{\mu_1^A,t} = p_{\mu_2^B,t} = 0$. Consider the strategy profile in period t: $\{p_{\mu_1^B,t}, p_{\mu_2^A,t}, p_{\mu_1^A,t} = p_{\mu_2^B,t} = 0\}$. Let ω represent some number in between 0 and 1. There are nine possible combinations of the values of $(p_{\mu_1^B,t}, p_{\mu_2^A,t})$: $(1,1)$, $(1,0)$, $(1,\omega)$, $(\omega,1)$, (ω,ω), $(\omega,0)$, $(0,1)$, $(0,\omega)$, $(0,0)$. Let

$$AW_{\mu_2} = U^A(\mu_2; h_t) - U^W(\mu_2; h_t; \{p_{\mu_1^B,t}, p_{\mu_2^A,t}, p_{\mu_1^A,t} = p_{\mu_2^B,t} = 0\})$$
$$WB_{\mu_1} = U^W(\mu_1; h_t; \{p_{\mu_1^B,t}, p_{\mu_2^A,t}, p_{\mu_1^A,t} = p_{\mu_2^B,t} = 0\}) - U^B$$

By continuity, we can find at least one of the following cases will exist:

2.2.1 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (1,1) \Longrightarrow AW_{\mu_2} > 0$, $WB_{\mu_1} < 0$

2.2.2 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (1,\omega) \Longrightarrow AW_{\mu_2} > 0$, $WB_{\mu_1} = 0$

2.2.3 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (1,0) \Longrightarrow AW_{\mu_2} > 0$, $WB_{\mu_1} > 0$

2.2.4 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (\omega,1) \Longrightarrow AW_{\mu_2} = 0$, $WB_{\mu_1} < 0$

2.2.5 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (\omega,\omega) \Longrightarrow AW_{\mu_2} = 0$, $WB_{\mu_1} = 0$

2.2.6 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (\omega,0) \Longrightarrow AW_{\mu_2} = 0$, $WB_{\mu_1} > 0$

2.2.7 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (0,1) \Longrightarrow AW_{\mu_2} < 0$, $WB_{\mu_1} < 0$

2.2.8 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (0,\omega) \Longrightarrow AW_{\mu_2} < 0$, $WB_{\mu_1} = 0$

2.2.9 $(p_{\mu_1^B,t}, p_{\mu_2^A,t}) = (0,0) \Longrightarrow AW_{\mu_2} < 0$, $WB_{\mu_1} > 0$

If any one of above cases exists, then the strategy profile in that case is an equilibrium strategy profile in period t.[15]

2.3 If $U^A(\mu_2; h_t) \geq U^A(\mu_1; h_t) > U^B$, $\{p_{\mu_1^B,t} = p_{\mu_2^B,t} = 0, p_{\mu_1^A,t} = p_{\mu_2^A,t} = 1\}$ is an equilibrium strategy profile in period t.

[15]Note, since the benefit from waiting is derived from the continuation game, we must construct the entire scheme of the continuation game first, then find out if the conjectured strategy profile $(p_{\mu_1^B,t}, p_{\mu_2^A,t})$ is indeed an equilibrium in period t. Same logic applies to the following proof.

$\vdots$

$\vdots$

Step K Continue to the subgame $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\}}(h_t)$ with all the possible types of individuals $\mu_1, \mu_2, \ldots, \mu_K$. That is, $M = \{\mu_1, \mu_2, \ldots, \mu_K\}$. There are $K+1$ possible cases:

K.1 If $U^B \geq U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t)$, then $\{p_{\mu_1^B,t} = p_{\mu_2^B,t} = \ldots = p_{\mu_K^B,t} = 1, p_{\mu_1^A,t} = p_{\mu_2^A,t} = \ldots = p_{\mu_K^A,t} = 0\}$ is an equilibrium strategy profile in period t.

$\vdots$

$\vdots$

K.K+1 If $U^A(\mu_K; h_t) \geq U^A(\mu_{K-1}; h_t) \geq \ldots \geq U^A(\mu_1; h_t) \geq U^B$, then $\{p_{\mu_1^B,t} = p_{\mu_2^B,t} = \ldots = p_{\mu_K^B,t} = 0, p_{\mu_1^A,t} = p_{\mu_2^A,t} = \ldots = p_{\mu_K^A,t} = 1\}$ is an equilibrium strategy profile in period t.

We can see in the final Step K if we replace h_t with h_1 then $\mathcal{G}_{\{\mu_1,\mu_2,\ldots,\mu_K\}}(h_1)$ is the original game.

(ii) Large Number and Patient Individuals:

If the number of individuals is large and $\mu_1^A(h_1)$ strictly smaller than $\overline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}^A}$ strictly greater than 0), by the Law of Large Numbers, the true value of the risky investment option A will be (approximately) revealed in the second period. Similarly, if the number of individuals is large and $\mu_1^B(h_1)$ strictly greater than $\underline{\mu}$ (finite discrete private signal space: $p_{\underline{\mu}^B}$ strictly greater than 0), by the Law of Large Numbers, the true value of the risky investment option A will be (approximately) revealed in the second period. In this case, if individuals are patient enough, then all individuals will wait in period 1 such that $\mu_1^A(h_1) = \overline{\mu}$ and $\mu_1^B(h_1) = \underline{\mu}$ (finite discrete private signal space: $p_{\overline{\mu}^A} = 0$ and $p_{\underline{\mu}^B} = 0$). This is a contradiction. Thus, if the number of individuals is large and individuals are patient enough, in any period $\infty > t > 1$, the game is "almost" the same as the period 1 game: either $\mu_t^A(h_t) = \mu_{t-1}^A(h_{t-1})$ or $\mu_t^A(h_t) \approx \mu_{t-1}^A(h_{t-1})$ (finite discrete private signal space: either $p_{\overline{\mu}^A} = 0$ or $p_{\overline{\mu}^A} \approx 0$); and either $\mu_t^B(h_t) = \mu_{t-1}^B(h_{t-1})$ or $\mu_t^B(h_t) \approx \mu_{t-1}^B(h_{t-1})$ (finite discrete private signal space: either $p_{\underline{\mu}^B} = 0$ or $p_{\underline{\mu}^B} \approx 0$). Thus, at any fixed time, there is a negligible proportion of individuals exercising A or B and so is the information disclosed. ■

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CHAPTER 3

Group Reputation — A Model of Corruption

3.1 Introduction

What is group reputation? How should its formation and evolution be modeled? The starting point of the reputation model is incomplete information, which induces either adverse selection, moral hazard, or both. Reputation matters when players want to establish a long-term relationship with others.

Tirole (1996) is the first attempt to model the idea of group reputation as an aggregate of individual reputations. Due to group pooling (individual players' unknown ages and imperfect signals of players' history records), individual reputations relate to group reputation; and the new members may suffer from the original sin of their elders. Levin (2001) adopts a similar idea that a player cannot be perfectly distinguished from others and argues that peers' past behaviors affect players' record of performance. Both papers focus on individual reputation and do not clarify the difference between individual reputation and group reputation.

In this paper, we define Individual Reputation and Group Reputation as follows:

A player A_i's **individual reputation** to do X with respect to some others P_j is the belief of P_j on the type or behavior of A_i to do X.[1]

Group G_k's **group reputation** to do X with respect to P_j is the belief of P_j on the type or behavior of any player $A_s \in G_k$, to whom P_j does not have individual information, to do X.

According to this definition, we divide group G_k into two separate subgroups: players whom P_j is familiar with (P_j has additional individual signals on these players), players whom P_j is not familiar with. For players belonging to the first subgroup, each player's individual reputation with respect to P_j may vary upon the individual signals P_j has. But for players belonging to the second subgroup, each player's individual reputation with respect to P_j is same as the group reputation because P_j does not have additional individual signals on these players.

For a sufficiently large group, it is safe to say that there are always some players within the group unfamiliar to some others P_j. If indeed P_j is familiar with everyone in a group G_k, we can define the group reputation of G_k with respect to P_j as follows: imagining if there were a player who belongs to G_k but P_j does not have individual information regarding this player, what is his individual reputation? And this represents the group reputation.

In other words, a player's group reputation is the belief others have about the characteristics of the group he belongs to, which is based only on group signals. A player's individual reputation is derived from his group reputation by adding individual signals.

In this paper, a model of group reputation of civil servants is constructed to identify the strategic behavior of potential bribers and civil servants, the cor-

[1] According to Hardin (1993), trust is a three-part relationship: A trust B to do X. Similarly, reputation is also a three-part relationship: B's reputation to do X with respect to A is A's belief on the type or behavior of B to do X.

responding levels of corruption, and possible anti-corruption policies along with their effects.

The definition of corruption according to Bardhan (1997) is “the use of public office for private gains, where an official (the agent) entrusted with carrying out a task by the public (the principal) engages in some sort of malfeasance for private enrichment which is difficult to monitor for the principal”. Most current literature on corruption focuses on the principal-agent relationship between officials and the government, in which the officials delegate the government to allocate some scarce resources.

In this paper, we focus on two types of corruption behavior of civil servants: accepting bribes and dereliction of duty. Civil servants have the right to examine and approve some project of the private agents by some criteria, such as the road test for a driver license. The civil servants could belong to the type of “good”, “bad”, or “opportunist”. The good type always rejects bribes and implements fair tests. The bad type always accepts bribes and intentionally places obstacles during the tests if there is no bribe. And the opportunist type will weigh the advantages and disadvantages to decide whether to accept bribes or intentionally place obstacles during the tests if there is no bribe. Since a private agent does not know the true type of a civil servant, he will decide whether or not to offer a bribe according to the current group reputation of the civil servants.

The reason to focus on these two types of corruption is that bribes accepted by civil servants are actually “protection money” to prevent them from dereliction of duty, which is different from the “grease money” as in the corruption on allocating scarce resources. The former is more closely linked to the civilians. And the result of this kind of corruption is much more severe because “protection money” directly affect the welfare of the civilians. The corruption related to “grease money" only affect the welfare of the civilians indirectly through embezzling the public resources by the officials and the bribers. In some cases, "grease money" even could reduce the inefficiency in public administration. For instance, Lui (1985) argues "the server could choose to speed up the services when briber is allowed" and as a result the outcome is socially optimal.

There are several related strands of literature. The first is on individual reputation. Holmstrom (1999) investigates the dynamic incentive problem – the agent has the strongest incentive to work hard to reveal his managerial ability. As time goes by, his ability is learned, and thus the reputation effect on incentive also decreases. Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989), Ely, Fudenberg and Levine (2004), and many others investigate the settings of a single long-run player and a sequence of short-run opponents – the long-run player tries to commit to some type to achieve highest possible utility. Horner (2002) introduces competition to keep high efforts sustainable.

The second is on statistical discrimination. Because agents cannot perfectly signal their characteristics, the multiplicity of equilibria becomes possible as the possibility of a differential treatment of agents based on some observable characteristics. Cornell and Welch (1996) develop a model on "screening discrimination" merely based on "unfamiliarity", which makes it more difficult to make accurate assessments. Fang (2001) shows that by allowing the firm to give preferential treatment to workers based on some "cultural activity", the society can partially overcome the informational free-riding problem. The critique on the statistical discrimination theory is that it is a static theory, which does not say much about reputation formation and its persistence.

For the dynamic reputation model, Diamond (1989) constructs a model in debt markets. His key point is that as time goes by, bad type drops out, which drives up the reputation for the remaining agents.

The paper is structured as follows. Section 3.2 describes the basic model and establishes the conditions for the possible steady states. Section 3.3 provides the dynamic analysis and studies the effectiveness of one time anti-corruption policy. Section 3.4 concludes.

3.2 Model

3.2.1 Basic Settings

In this section, we develop a model in which there exist a benevolent government, a group of social servants, and a large number of private agents. The benevolent government selects and supervises social servants who delegate the government to examine and approve some projects of the private agents by some criterion.

The civil servants could be the type of "good", "bad", or "opportunist", denoted as type "G", "B", "BG" respectively. The good type "G" always rejects bribes and implement fair tests. The bad type "B" always accepts bribes if there are any and intentionally place obstacles during the tests if there is no bribe. And the opportunist type "BG" will weigh the advantage and disadvantage to decide whether to accept bribes or intentionally place obstacles during the tests if there is no bribe. Because the behavior for the type "G", "B" is fixed, we only need to study the strategic behavior of the "opportunist" type "BG".

If a civil servant accepts a bribe, there is probability $\alpha \in (0, 1)$ he will be detected and removed from the office by the government. If a civil servant intentionally places obstacles during a test, there is probability $\gamma \in (0, 1)$ he will be detected and removed from the office by the government. Thus, there are two types of corruption behavior for the civil servants: accepting bribes and dereliction of duty. And (α, γ) represents the supervision effort level of the government regarding to these two types of corruption behavior.

The civil servants alive in date t remain in the economy in date $t+1$ with probability $\lambda \in (0, 1)$. We assume that each quit is offset by the arrival of a new civil servant selected by the government from a population with proportion of the three types "G", "B", "BG": f_G, f_B, f_{BG}. So the size of the civil servants remains constant.

At the beginning of each period, a number of private agents is selected by the government to get their projects tested. Each private agent included in the tests will decide to offer a bribe or not to the civil servant who is assigned to test his project. Then the civil servants will decide to reject or accept bribes if there are any. If there is no bribe, the civil servants will decide to implement fair tests or intentionally place obstacles during the tests. The timing of the model in any arbitrary period t is summarized in the figure below.

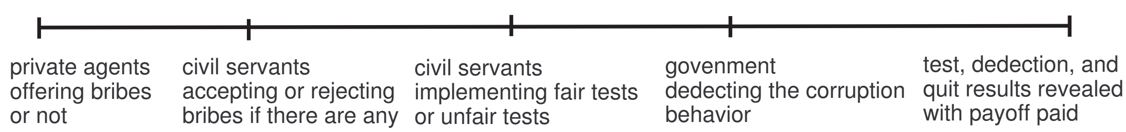

Figure 3.1: Timing in period t

In period t, the utility of each private agent included in the tests from offering a bribe and not offering a bribe are as follows:

$$U_t^b = P_{A,t}[\mu_G(1-\alpha)X - C] + (1 - P_{A,t})[\mu_G X - \eta C]$$

$$U_t^n = P_{B,t}[\mu_B X] + (1 - P_{B,t})[\mu_G X]$$

where $P_{A,t}$ is the belief of the private agent that the civil servant he meets will accept a bribe if there is any; and $P_{B,t}$ is the belief of the private agent that the civil servant he meets will intentionally place obstacles during the test if there is no bribe.[2] μ_G is the probability of the project being approved under a fair test. μ_B is the probability of the project being approved under an unfair test. X is benefit from an approved project. C is the cost of bribe.[3] $\eta \in (0,1)$ is the share of loss on a bribe if it is rejected.

So, the private agent will not offer a bribe at the beginning of period t if $U_t^b \leq U_t^n$. That is,

$$P_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + P_{A,t}[(1-\eta)C + \alpha\mu_G X] \tag{3.1}$$

In period t, if there is a bribe, the utility of the "opportunist" type "BG" civil servant from rejecting it and accepting it are as follows:

$$V_t^R = Y + \delta\lambda V_{t+1}$$
$$V_t^A = Y + C + \delta(1-\alpha)\lambda V_{t+1} - \Gamma(P_{A,t})$$

where Y is the wage of the civil servant in each period. $\delta \in (0,1)$ is the discount factor. V_{t+1} is the continuation payoff in period $t+1$. $\Gamma(P_{A,t})$ is the cost from accepting a bribe, which is a decreasing function of $P_{A,t}$.[4]

So, the "opportunist" type "BG" civil servant will reject bribes in period t if $V_t^R \geq V_t^A$. That is,

$$\delta\alpha\lambda V_{t+1} \geq C - \Gamma(P_{A,t}) \tag{3.2}$$

The utility of the "opportunist" type "BG" civil servant in period t from implementing a fair test or intentionally placing obstacles during the test if there

[2] $\{P_{A,t}, P_{B,t}\}$ represents the group reputation of the civil servants in period t, which is the belief of the private agents on the two types of corruption behavior of the civil servants: accepting bribes and dereliction of duty.

[3] Here, we assume the size of bribe fixed. Later, we may incorporate the endogenous size of bribe.

[4] We assume that the "opportunist" type "BG" civil servant who accepts bribes will suffer some cost. It may be due to the secrecy of bribery behavior and mental burden of pursuing private gains by using public office. And this cost will decrease as accepting bribe becomes a general mood of the society.

is no bribe are as follows:

$$V_t^G = Y + \delta\lambda V_{t+1}$$

$$V_t^B = Y + \delta(1-\gamma)\lambda V_{t+1}$$

Since $V_t^G \geq V_t^B$, the "opportunist" type "BG" civil servant will always implement fair tests no matter the private agent offers bribes or not. The logic behind is that even though the "opportunist" type "BG" civil servants may accept bribes, they are still not so "bad" as the bad type "B" civil servants are. They are not willing to harm others while not benefit themselves.

In this paper, we focus on the symmetric equilibrium. For simplicity, we assume that the number of civil servants and private agents are so large that in each period the pairs of the civil servants and private agents who have matched before are relatively small. Thus, the effect of re-match could be omitted upon updating the private agents' belief of the entire group of civil servants, which is the group reputation for the entire group of civil servants.[5] If indeed re-match occurs, then the private agent in this re-match will update the belief on the civil servant in this re-match based on the current group belief and the history record of this civil servant, which is the individual reputation of this civil servant.

Now, we need to identify the evolution of proportions of the three types of civil servants as time goes by. Denote $f_{G,t}$, $f_{B,t}$, $f_{BG,t}$ as the fractions of "G", "B", "BG" type of civil servants respectively in period t. Then $\{f_{B,t}, f_{BG,t}, f_{G,t}\}$ represents the state of the economy in period t. In period $t+1$, the transition of the state of the economy is described in the following three cases, depending on the actions chosen by the private agents and the "opportunist" type "BG" civil servants in period t.

[5]In other words, the setting of our model is equivalent to the setting of a group of long-run players and a sequence of short-run opponents.

Case 1: private agents NOT offering bribes in period t

$$\begin{aligned} f_{G,t+1} &= \lambda f_{G,t} + [(1-\lambda) + \lambda\gamma f_{B,t}] f_G \\ f_{BG,t+1} &= \lambda f_{BG,t} + [(1-\lambda) + \lambda\gamma f_{B,t}] f_{BG} \\ f_{B,t+1} &= \lambda(1-\gamma) f_{B,t} + [(1-\lambda) + \lambda\gamma f_{B,t}] f_B \end{aligned} \tag{3.3}$$

Case 2: private agents offering bribes and the "opportunist" type "BG" civil servants rejecting the bribes in period t

$$\begin{aligned} f_{G,t+1} &= \lambda f_{G,t} + [(1-\lambda) + \lambda\alpha f_{B,t}] f_G \\ f_{BG,t+1} &= \lambda f_{BG,t} + [(1-\lambda) + \lambda\alpha f_{B,t}] f_{BG} \\ f_{B,t+1} &= \lambda(1-\alpha) f_{B,t} + [(1-\lambda) + \lambda\alpha f_{B,t}] f_B \end{aligned} \tag{3.4}$$

Case 3: private agents offering bribes and the "opportunist" type "BG" civil servants accepting the bribes in period t

$$\begin{aligned} f_{G,t+1} &= \lambda f_{G,t} + [(1-\lambda) + \lambda\alpha(f_{B,t} + f_{BG,t})] f_G \\ f_{BG,t+1} &= \lambda(1-\alpha) f_{BG,t} + [(1-\lambda) + \lambda\alpha(f_{B,t} + f_{BG,t})] f_{BG} \\ f_{B,t+1} &= \lambda(1-\alpha) f_{B,t} + [(1-\lambda) + \lambda\alpha(f_{B,t} + f_{BG,t})] f_B \end{aligned} \tag{3.5}$$

Since only the "bad" type "B" civil servant will intentionally place obstacles during the tests if there is no bribe, $P_{B,t} = f_{B,t}$. For the symmetric equilibrium, either only the "bad" type "B" civil servant will accept bribes or both the "bad" type "B" civil servant and the "opportunist" type "BG" civil servant will accept bribes if there are any. That is to say, either $P_{A,t} = f_{B,t}$ or $P_{A,t} = f_{B,t} + f_{BG,t}$.

3.2.2 Steady States

In this section, we analyze the four possible steady states and their feasible conditions.

3.2.2.1 Low Corruption Steady State I (LCSS-I)

The first one is Low Corruption Steady State I (LCSS-I), in which the private agents do not offer bribes and the "opportunist" type "BG" civil servants reject bribes if there are any. By equation 3.3, we can derive the proportions of three types of civil servants at LCSS-I, denoted as $\underline{f_G^I}$, $\underline{f_B^I}$, $\underline{f_{BG}^I}$.

$$\underline{f_G^I} = \frac{1-\lambda+\lambda\gamma}{1-\lambda+\lambda\gamma(1-f_B)} f_G$$
$$\underline{f_{BG}^I} = \frac{1-\lambda+\lambda\gamma}{1-\lambda+\lambda\gamma(1-f_B)} f_{BG}$$
$$\underline{f_B^I} = \frac{1-\lambda}{1-\lambda+\lambda\gamma(1-f_B)} f_B$$

The utility for the "opportunist" type "BG" civil servant at LCSS-I, denoted as V_L, is

$$V_L = Y + \delta\lambda V_L \Longrightarrow V_L = \frac{1}{1-\delta\lambda} Y$$

At LCSS-I, $P_{B,t} = P_{A,t} = \underline{f_B^I}$. Back to inequality 3.1 and 3.2, to induce a private agent not to offer a bribe and an "opportunist" type "BG" civil servant reject a bribe if there is any, the following conditions must hold.

Feasible Conditions of LCSS-I:

$$(1-\delta\lambda)\Gamma(\underline{f_B^I}) \geq (1-\delta\lambda)C - \delta\alpha\lambda Y$$
$$\underline{f_B^I}[(\mu_G-\mu_B)X] \leq \eta C + \underline{f_B^I}[(1-\eta)C + \alpha\mu_G X]$$

3.2.2.2 Low Corruption Steady State II (LCSS-II)

The second steady state is Low Corruption Steady State II (LCSS-II), in which the private agents do not offer bribes and the "opportunist" type "BG" civil servants accept bribes if there are any. By equation 3.3, we can derive the proportions of three types of civil servants at LCSS-II. Because the private agents do not offer bribes, the proportions of three types of civil servants at LCSS-II are same as the proportions of three types of civil servants at LCSS-I.

Same logic, the utility for the "opportunist" type "BG" civil servant at LCSS-II is same as the utility for the "opportunist" type "BG" civil servant at LCSS-I, V_L.

At LCSS-II, $P_{B,t} = \underline{f_B^I}$, and $P_{A,t} = \underline{f_B^I} + \underline{f_{BG}^I}$. Back to inequality 3.1 and 3.2, to induce a private agent not to offer a bribe and an "opportunist" type "BG" civil servant accept a bribe if there is any, the following conditions must hold.

Feasible Conditions of LCSS-II:

$$(1-\delta\lambda)\Gamma(\underline{f_B^I} + \underline{f_{BG}^I}) < (1-\delta\lambda)C - \delta\alpha\lambda Y$$
$$\underline{f_B^I}[(\mu_G - \mu_B)X] \leq \eta C + (\underline{f_B^I} + \underline{f_{BG}^I})[(1-\eta)C + \alpha\mu_G X]$$

3.2.2.3 Low Corruption Steady State III (LCSS-III)

The third steady state is Low Corruption Steady State III (LCSS-III), in which the private agents offer bribes and the "opportunist" type "BG" civil servants reject bribes if there are any. By equation 3.3, we can derive the proportions of three types of civil servants at LCSS-III, denoted as $\underline{f_G^{III}}$, $\underline{f_B^{III}}$, $\underline{f_{BG}^{III}}$.

$$\underline{f_G^{III}} = \frac{1-\lambda+\lambda\alpha}{1-\lambda+\lambda\alpha(1-f_B)} f_G$$
$$\underline{f_{BG}^{III}} = \frac{1-\lambda+\lambda\alpha}{1-\lambda+\lambda\alpha(1-f_B)} f_{BG}$$
$$\underline{f_B^{III}} = \frac{1-\lambda}{1-\lambda+\lambda\alpha(1-f_B)} f_B$$

Due the the rejection of the bribe, the utility for the "opportunist" type "BG" civil servant at LCSS-III is the same as the utility for the "opportunist" type "BG" civil servant at LCSS-I and LCSS-II, V_L.

At LCSS-III, $P_{B,t} = P_{A,t} = \underline{f_B^{III}}$. Back to inequality 3.1 and 3.2, to induce a private agent offer a bribe and an "opportunist" type "BG" civil servant reject a bribe if there is any, the following conditions must hold.

Feasible Conditions of LCSS-III:

$$(1-\delta\lambda)\Gamma(\underline{f_B^{III}}) \geq (1-\delta\lambda)C - \delta\alpha\lambda Y$$
$$\underline{f_B^{III}}[(\mu_G-\mu_B)X] > \eta C + \underline{f_B^{III}}[(1-\eta)C + \alpha\mu_G X]$$

3.2.2.4 High Corruption Steady State (HCSS)

The last possible steady state is High Corruption Steady State (HCSS), in which the private agents offer bribes and the "opportunist" type "BG" civil servants accept bribes if there are any. By equation 3.3, we can derive the proportions of three types of civil servants at HCSS, denoted as $\overline{f_G}$, $\overline{f_{BG}}$, $\overline{f_B}$.

$$\overline{f_G} = \frac{1-\lambda+\lambda\alpha}{1-\lambda+\lambda\alpha(1-f_B-f_{BG})} f_G$$
$$\overline{f_{BG}} = \frac{1-\lambda}{1-\lambda+\lambda\alpha(1-f_B-f_{BG})} f_{BG}$$
$$\overline{f_B} = \frac{1-\lambda}{1-\lambda+\lambda\alpha(1-f_B-f_{BG})} f_B$$

At HCSS, $P_{B,t} = \overline{f_B}$, and $P_{A,t} = \overline{f_{BG}} + \overline{f_B}$. The utility for the "opportunist" type "BG" civil servant at HCSS, denoted as V_H, is

$$V_H = Y + C - \Gamma(\overline{f_{BG}} + \overline{f_B}) + \delta\lambda(1-\alpha)V_H$$
$$\Longrightarrow V_H = \frac{1}{1-\delta\lambda(1-\alpha)}(Y + C - \Gamma(\overline{f_{BG}} + \overline{f_B}))$$

Back to inequality 3.1 and 3.2, to induce a private agent to offer a bribe and an "opportunist" type "BG" civil servant accept a bribe if there is any, the following conditions must hold.

Feasible Conditions of HCSS:

$$(1-\delta\lambda)\Gamma(\overline{f_{BG}} + \overline{f_B}) < (1-\delta\lambda)C - \delta\alpha\lambda Y$$
$$\overline{f_B}[(\mu_G-\mu_B)X] > \eta C + (\overline{f_{BG}} + \overline{f_B})[(1-\eta)C + \alpha\mu_G X]$$

3.3 Dynamic Analysis and Anti-Corruption

In this section, we analyze the dynamical situation if the economy in period t is currently at some arbitrary state: $\{f_{B,t}, f_{BG,t}, f_{G,t}\}$. Then we discuss the effectiveness of possible anti-corruption policies.

3.3.1 Dynamic Analysis

Proposition 3.1 *Suppose in period t the economy is currently at some state: $\{f_{B,t}, f_{BG,t}, f_{G,t}\}$. There are only four possible areas of the state space.*

> ***Low corruption area I (L-I):*** *in period t, private agents will not offer bribes and the "opportunist" type "BG" civil servants will reject bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3.*
>
> ***Low corruption area II (L-II):*** *in period t, private agents will not offer bribes and the "opportunist" type "BG" civil servants will accept bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3.*
>
> ***Low corruption area III (L-III):*** *in period t, private agents will offer bribes and the "opportunist" type "BG" civil servants will reject bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.4.*
>
> ***High corruption area (H):*** *in period t, private agents will offer bribes and the "opportunist" type "BG" civil servants will accept bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.5.*

i) *If* $f_{B,t} \leq \min\{f_B^*, f_B^{**}\}$*,* $f_{B,t} \in$ *L-I.*

ii) *If* $f_B^* < f_{B,t} \leq f_B^{**}$*,* $f_{B,t} \in$ *L-III.*

iii) *If* $f_B^{**} < f_{B,t} \leq f_B^*$ *and* $f_{BG,t} \geq f_{BG}^*(f_{B,t})$*,* $f_{B,t} \in$ *L-I or L-II.*

iv) *If* $\max\{f_B^*, f_B^{**}\} < f_{B,t}$ *and* $f_{BG,t} \geq f_{BG}^*(f_{B,t})$*,* $f_{B,t} \in$ *L-II or L-III.*

v) *If* $f_{B,t} > f_B^{**}$ *and* $f_{BG,t} < f_{BG}^*(f_{B,t})$*,* $f_{B,t} \in$ *or L-III or H.*

where $f_B^*, f_B^{**}, f_{BG}^*(f_B)$ *are the solutions of the following equations.*[6]

$$f_B^*[(\mu_G - \mu_B)X] = \eta C + f_B^*[(1-\eta)C + \alpha\mu_G X] \tag{3.6}$$

$$(1-\delta\lambda)\Gamma(f_B^{**}) = (1-\delta\lambda)C - \delta\alpha\lambda Y \tag{3.7}$$

$$f_{BG}^*(f_B) = -\frac{\eta C}{(1-\eta)C + \alpha\mu_G X} + \frac{(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X]}{(1-\eta)C + \alpha\mu_G X} f_B \tag{3.8}$$

Proof. See the Appendix. ■

Figure 3.2 sketches out the state space partition of Proposition 3.1 in the case of $f_B^{**} > f_B^*$ and $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] > 0$.[7]

Figure 3.3 sketches out the state space partition of Proposition 3.1 in the case of $f_B^{**} \leq f_B^*$ and $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] > 0$.

Figure 3.4 sketches out the state space partitions of Proposition 3.1 in the case of $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] \leq 0$. In this case, both Low corruption area III (L-III) and High corruption area (H) disappear.[8]

[6] f_B^* *and* f_B^{**} *must be in between* 0 *and* 1*. If the solutions of equation 3.6 and/or equation 3.7 out of this range, we say* f_B^* *and/or* f_B^{**} *do not exist.*

[7] Note, in this case, $f_{BG}^*(f_B^*) = 0$. Thus, the extended line of $f_{BG}^*(f_{B,t})$ will go through the point $(f_{B,t} = f_B^*, f_{BG,t} = 0)$.

[8] In this case, f_B^* is negative. Since the solution of equation 3.6 has negative solution, this means private agents never offer bribes.

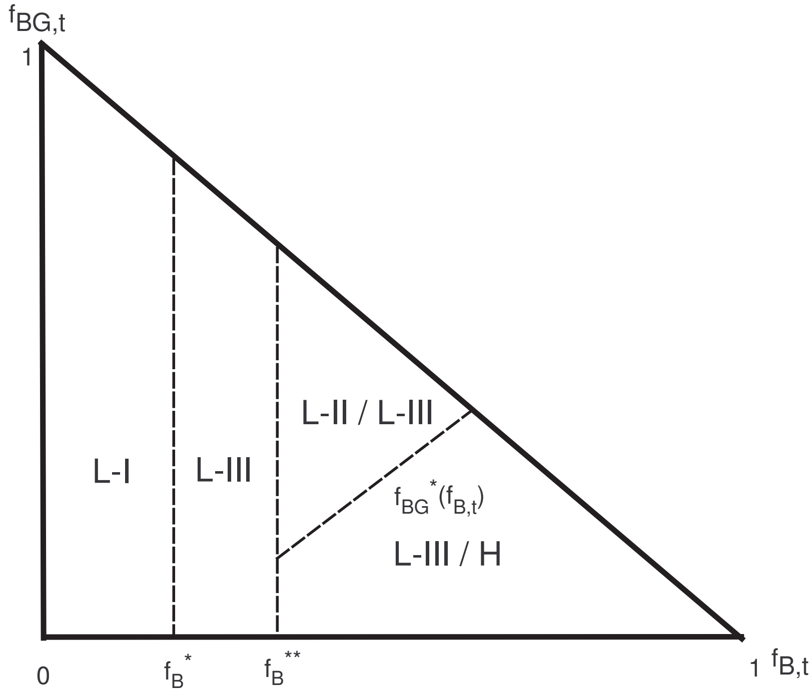

Figure 3.2: $f_B^{**} > f_B^*$ and $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] > 0$

There are some more minor variations of the state space partitions depending on the values of $f_B^*, f_B^{**}, f_{BG}^*(f_B)$. But the basic shapes are described as in figure 3.2 to 3.4.

After discussing the transition of the state in period $t+1$, the natural extension is to characterize the long run properties, that is, whether the economy can converge to some steady state. From proposition 3.1, we have the following corollary.

Corollary 3.1 *Low Corruption Steady State I (LCSS-I) is feasible if* $\{\underline{f_G^I}, \underline{f_B^I}, \underline{f_{BG}^I}\}$ *is in the Low corruption area I (L-I). Similarly, Low Corruption Steady State II (LCSS-II) is feasible if* $\{\underline{f_G^I}, \underline{f_B^I}, \underline{f_{BG}^I}\}$ *is in the Low corruption area II (L-II); Low Corruption Steady State III (LCSS-III) is feasible if* $\{\underline{f_G^{III}}, \underline{f_B^{III}}, \underline{f_{BG}^{III}}\}$ *is in the Low corruption area III (L-III); High Corruption Steady State (HCSS) is feasible*

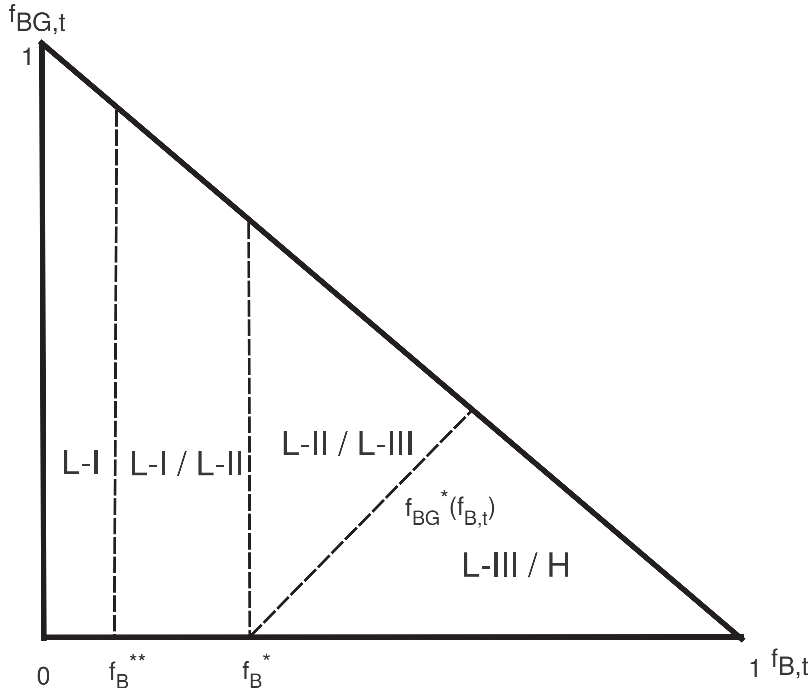

Figure 3.3: $f_B^{**} \leq f_B^*$ and $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] > 0$

if $\{\overline{f_G}, \overline{f_{BG}}, \overline{f_B}\}$ *is in the High corruption area (H).*

Further, if a steady state is feasible and the economy is currently at some state in the same area of this steady state, the economy will converge to this steady state.

Proof. See the Appendix. ■

In the long run, if no steady state is feasible, then the state of the economy will fluctuate back and forth among these four possible areas of state space. Even in the case that some steady state is feasible, the state of the economy may not converge to it.

For instance, suppose the state space partition is described in figure 3.1 and Low Corruption Steady State I (LCSS-I) is in the the Low corruption area III (L-

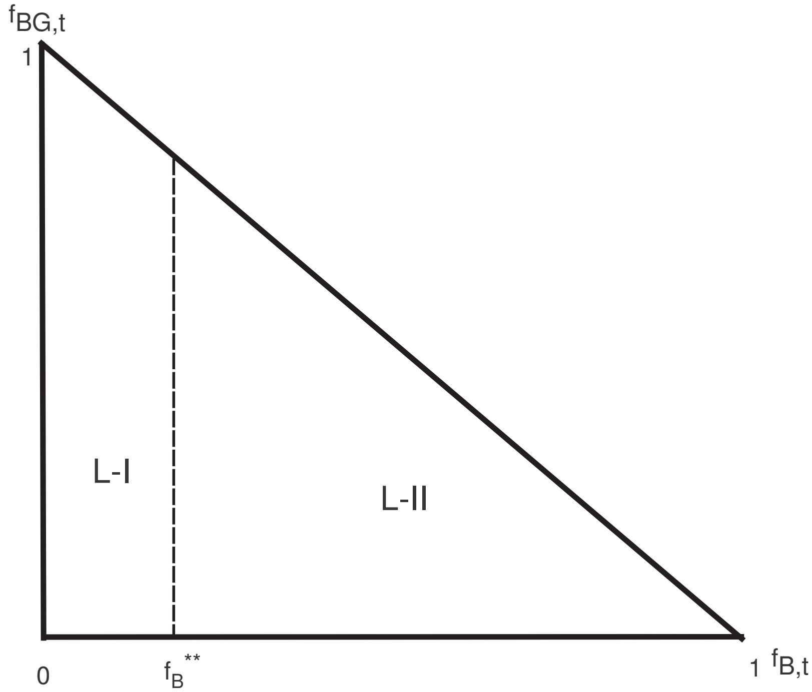

Figure 3.4: $(\mu_G - \mu_B)X - [(1-\eta)C + \alpha\mu_G X] \leq 0$

III) and Low Corruption Steady State III (LCSS-III) is in the the Low corruption area I (L-I). In period t, if $\{f_{B,t}, f_{BG,t}, f_{G,t}\}$ is in the Low corruption area I (L-I), the transition of the state of the economy from period t to period $t+1$ still follows equations 3.3 and it will be on the path of converging to the Low Corruption Steady State I (LCSS-I). But once it crosses the boundary of the Low corruption area I (L-I) and goes into the Low corruption area III (L-III), the transition of the state of the economy will follow equations 3.4 and will be on the path of converging to the Low Corruption Steady State III (LCSS-III) and go back to the Low corruption area I (L-I). The state of the economy will fluctuate back and forth between the Low corruption area I (L-I) and the Low corruption area III (L-III). In this case, if High Corruption Steady State (HCSS) is in the High corruption area (H), the state of the economy may not converge to it even though it is feasible. Thus, we have the following corollary.

Corollary 3.2 *In the long run, the state of the economy may not converge to any steady state, even in the case that some steady state is feasible.*

3.3.2 One Time Anti-Corruption

In this section, we assume that the economy currently suffers from high level corruption, i.e., the economy is at the High Corruption Steady State (HCSS) or fluctuating in between the High corruption area (H) and some Low corruption area. The government introduces a one time anti-corruption policy, aiming to lead to a low corruption level permanently.

One time anti-corruption policy means a combination of new level of supervision effort $\{\alpha_t, \gamma_t\}$ in period t. And it only lasts one period. After period t, the supervision effort goes back to the original level. We say a one time anti-corruption policy is effective if after period t the economy converges to some Low Corruption Steady State or fluctuates in between some Low corruption areas.

Proposition 3.2 *One time anti-corruption policy may or may not be effective depending on the environment of the economy.*

i) *In the case when there does not exist one time anti-corruption policy to effectively turn around the high level corruption, the government must introduce a permanent anti-corruption policy, i.e., permanently adjusting the level of supervision effort.*

ii) *In the case when there exists one time anti corruption policy to effectively turn around the high level corruption, when the government sets a one time anti-corruption policy, not only does it have to increase the supervision* *effort on detecting the bribery behavior (α), but also it needs to consider the the supervision effort on detecting the behavior of intentionally placing obstacles (dereliction of duty) during the test (γ). Anti-corruption should work along both lines.*

Proof. See the Appendix. ■

3.3.3 Re-match

Since we assume that the number of civil servants and private agents are so large that in each period the pairs of the civil servants and private agents who have matched before are relatively small. Thus, the effect of re-match could be omitted upon updating the private agents' belief of the entire group of civil servants, which is the group reputation for the entire group of civil servants as we have discussed so far.

If indeed re-match occurs, then the private agent in this re-match will update the belief on the civil servant in this re-match based on the current group belief and the history record of this civil servant, which is the individual reputation of this civil servant.

For instance, if the the civil servant has rejected this private agent's bribe before, the private agent will not offer a bribe because he knows that this civil servant is "G" type. If the civil servant has intentionally placed obstacles during the test before, the private agent knows that this civil servant is "B" type. He is more likely to offer a bribe to keep this civil servant from intentionally placing obstacles during the test.

3.4 Conclusion

This paper presents a group reputation model of corruption. First, we define the group reputation and individual reputation. Then, a model of group reputation of civil servants is constructed to identify the strategic behavior of potential bribers and civil servants. We identify four possible steady states and their feasible conditions, provide dynamic analysis and study the effectiveness of one time anti-corruption policy. We show that one time anti-corruption policy may or may not be effective in successfully overturning the high corruption steady state depending on the economic environment.

In the case when there does not exist one time anti-corruption policy to effectively turn around the high level corruption, the government must introduce a permanent anti-corruption policy, i.e., permanently adjusting the level of supervision effort.

In the case when there exists one time anti corruption policy to effectively turn around the high level corruption, then when the government sets a one time anti-corruption policy, not only does it have to increase the supervision effort on detecting the bribery behavior (α), but also it needs to consider the the supervision effort on detecting the behavior of intentionally placing obstacles (dereliction of duty) during the test (γ). Anti-corruption should work along both lines.

Finally, we assume the effect of re-match could be omitted upon updating the private agents' belief of the entire group of civil servants. This simplifies the model a lot. If we relax this assumption, we may get much richer dynamic scenarios on the interactions between group reputation and individual reputation.

3.5 Appendix

Proof of Proposition 3.1

i) If $f_{B,t} \leq \min\{f_B^*, f_B^{**}\}$, $f_{B,t} \in$ L-I.

Since the continuation payoff of the civil servants is bounded below by $V_L = \frac{1}{1-\delta\lambda}Y$, if $f_{B,t} \leq \min\{f_B^*, f_B^{**}\}$, by equation 3.7, we have

$$\delta\alpha\lambda V_{t+1} \geq \delta\alpha\lambda\frac{1}{1-\delta\lambda}Y = C - \Gamma(f_B^{**}) \geq C - \Gamma(f_{B,t})$$

By inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will reject bribes if there are any.[9] By equation 3.6, we have

$$f_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + f_{B,t}[(1-\eta)C + \alpha\mu_G X]$$

By inequality 3.1, private agents in period t will not offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3.

ii) If $f_B^* < f_{B,t} \leq f_B^{**}$, $f_{B,t} \in$ L-III.

Similar to the proof before, since the continuation payoff of the civil servants is bounded below by $V_L = \frac{1}{1-\delta\lambda}Y$, if $f_B^* < f_{B,t} \leq f_B^{**}$, by equation 3.7 we have

$$\delta\alpha\lambda V_{t+1} \geq \delta\alpha\lambda\frac{1}{1-\delta\lambda}Y = C - \Gamma(f_B^{**}) \geq C - \Gamma(f_{B,t})$$

By inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will reject bribes if there are any.[10]

By equation 3.6, we have

$$f_{B,t}[(\mu_G - \mu_B)X] > \eta C + f_{B,t}[(1-\eta)C + \alpha\mu_G X]$$

[9] Similar to the multiple equilibria issue in the coordination game, there could be another equilibrium, in which the "opportunist" type "BG" civil servants will accept bribes if there are any.

[10] Same as the multiple equilibria issue in the coordination game, there could be another equilibrium, in which the "opportunist" type "BG" civil servants will accept bribes if there are any.

By inequality 3.1, private agents in period t will offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.4.

iii) If $f_B^{**} < f_{B,t} \leq f_B^*$ and $f_{BG,t} \geq f_{BG}^*(f_{B,t})$, $f_{B,t} \in$ L-I or L-II.

Since $f_B^{**} < f_{B,t}$, we do not have a definite answer whether the "opportunist" type "BG" civil servants will accept or reject bribes in period t. There are two possible situations.

First, if the continuation payoff of the civil servants V_{t+1} is small such that

$$\delta\alpha\lambda V_{t+1} < C - \Gamma(f_{B,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will accept bribes if there are any. If $f_{BG,t} \geq f_{BG}^*(f_{B,t})$, by equation 3.8, we have

$$f_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + (f_{B,t} + f_{BG,t})[(1-\eta)C + \alpha\mu_G X]$$

By inequality 3.1, private agents in period t will not offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3. Thus, in this case, $f_{B,t} \in$ L-II.

Second, if the continuation payoff of the civil servants V_{t+1} is large such that

$$\delta\alpha\lambda V_{t+1} \geq C - \Gamma(f_{B,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will reject bribes if there are any. If $f_{B,t} \leq f_B^*$, by equation 3.6, we have

$$f_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + f_{B,t}[(1-\eta)C + \alpha\mu_G X]$$

By inequality 3.1, private agents in period t will not offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3. Thus, in this case, $f_{B,t} \in$ L-I.

iv) If $\max\{f_B^*, f_B^{**}\} < f_{B,t}$ **and** $f_{BG,t} \geq f_{BG}^*(f_{B,t})$**,** $f_{B,t} \in$ **L-II or L-III.**

Similar to the proof before, since $f_B^{**} < f_{B,t}$, we do not have a definite answer whether the "opportunist" type "BG" civil servants will accept or reject bribes in period t. There are two possible situations.

First, if the continuation payoff of the civil servants V_{t+1} is small such that

$$\delta\alpha\lambda V_{t+1} < C - \Gamma(f_{B,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will accept bribes if there are any. If $f_{BG,t} \geq f_{BG}^*(f_{B,t})$, by equation 3.8, we have

$$f_{B,t}[(\mu_G - \mu_B)X] \leq \eta C + (f_{B,t} + f_{BG,t})[(1-\eta)C + \alpha\mu_G X]$$

By inequality 3.1, private agents in period t will not offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.3. Thus, in this case, $f_{B,t} \in$ L-II.

Second, if the continuation payoff of the civil servants V_{t+1} is large such that

$$\delta\alpha\lambda V_{t+1} \geq C - \Gamma(f_{B,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will reject bribes if there are any. If $f_{B,t} > f_B^*$, by equation 3.6, we have

$$f_{B,t}[(\mu_G - \mu_B)X] > \eta C + f_{B,t}[(1-\eta)C + \alpha\mu_G X]$$

By inequality 3.1, private agents in period t will offer bribes. The transition of the state of the economy from period t to period $t+1$ follows equations 3.4. Thus, in this case, $f_{B,t} \in$ L-III.

v) If $f_{B,t} > f_B^{**}$ **and** $f_{BG,t} < f_{BG}^*(f_{B,t})$**,** $f_{B,t} \in$ **L-III or H.**

If $f_{BG,t} < f^*_{BG}(f_{B,t})$, by equation 3.8, we have

$$\begin{aligned} f_{B,t}[(\mu_G - \mu_B)X] &> \eta C + (f_{B,t} + f_{BG,t})[(1-\eta)C + \alpha\mu_G X] \\ &\geq \eta C + f_{B,t}[(1-\eta)C + \alpha\mu_G X] \end{aligned}$$

By inequality 3.1, private agents in period t will offer bribes.

Since $f^{**}_B < f_{B,t}$, we do not have a definite answer whether the "opportunist" type "BG" civil servants will accept or reject bribes in period t. There are two possible situations.

First, if the continuation payoff of the civil servants V_{t+1} is small such that

$$\delta\alpha\lambda V_{t+1} < C - \Gamma(f_{B,t} + f_{BG,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will accept bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.5. Thus, in this case, $f_{B,t} \in$ H.

Second, if the continuation payoff of the civil servants V_{t+1} is large such that

$$\delta\alpha\lambda V_{t+1} \geq C - \Gamma(f_{B,t} + f_{BG,t})$$

by inequality 3.2, this means in period t the "opportunist" type "BG" civil servants will reject bribes if there are any. The transition of the state of the economy from period t to period $t+1$ follows equations 3.4. Thus, in this case, $f_{B,t} \in$ L-III. ■

Proof of Corollary 3.1

From section 3.2.2.1, we have $\{\underline{f^I_G}, \underline{f^I_B}, \underline{f^I_{BG}}\}$. If Low Corruption Steady State I (LCSS-I) is in the the Low corruption area I (L-I), we can easily check that the feasible conditions of Low Corruption Steady State I (LCSS-I) described in section 3.2.2.1 are satisfied.

In L-I, the transition of the state of the economy follows equations 3.3. In period $s+1$, where $s \geq t+1$

$$\begin{aligned} f_{B,s+1} &= \lambda(1-\gamma)f_{B,s} + [(1-\lambda) + \lambda\gamma f_{B,s}]f_B \\ &= (1-\lambda)f_B + \lambda[1-\gamma(1-f_B)]f_{B,s} \end{aligned}$$

Since $\lambda[1-\gamma(1-f_B)] < 1$, $f_{B,s+1} < f_{B,s}$ if $f_{B,s} > \underline{f_B^I}$; $f_{B,s+1} > f_{B,s}$ if $f_{B,s} < \underline{f_B^I}$; $f_{B,s+1} = f_{B,s} = \underline{f_B^I}$ if $f_{B,s} = \underline{f_B^I}$. Thus, $f_{B,s}$ will converge to $\underline{f_B^I}$. Once $f_{B,s}$ converges to $\underline{f_B^I}$, by equations 3.3, similarly $f_{G,s}$ and $f_{BG,s}$ will converge to $\underline{f_G^I}$ and $\underline{f_{BG}^I}$ respectively. This means if a steady state is feasible and the economy is currently at some state in the same area of this steady state, the economy is on the LCSS-I path and will monotonously converge to LCSS-I.

Similarly, we can check the feasible conditions for all others steady states if they are in the corresponding areas and the convergence property if the economy is at some state in the same area of the state states. ■

Proof of Proposition 3.2

i) For instance, suppose the state space partition is described in figure 3.1 and all the steady states are in the High corruption area (H). Thus, High Corruption Steady State (HCSS) is feasible and no matter where the state of the economy is, the economy will converge to High Corruption Steady State (HCSS). In this case, any one time anti-corruption policy never works. It only can low the level of corruption for a period of time, then the corruption level will increase back. The government must permanently adjust the level of supervision effort to change the environment of the economy.

ii) For instance, suppose the state space partition is described in figure 3.1 without Low corruption area II (L-II) and Low Corruption Steady State I (LCSS-I) is in the Low corruption area I (L-I), Low Corruption Steady State III (LCSS-

III) and High Corruption Steady State (HCSS) in the High corruption area (H). Thus, Low Corruption Steady State I (LCSS-I) and High Corruption Steady State (HCSS) are feasible. If the the state of the economy is in the Low corruption area I (L-I), it will converge to Low Corruption Steady State I (LCSS-I). Otherwise, it will converge to High Corruption Steady State (HCSS). Therefore, to let a one time anti-corruption policy in some period t effective, the state of the economy in period $t+1$ must be in the Low corruption area I (L-I), i.e., $f_{B+1} \leq f_B^*$, where where f_B^* are the solutions of the equation 3.6.

Suppose the economy is currently at HCSS, then the sufficient condition to let the one time anti-corruption successfully covert the economy from HCSS to LCSS-I is $\alpha_t \geq \alpha_I^*$ and $\gamma_t \geq \gamma_I^*$, where α_I^*, γ_I^* are the solutions of following equations.

$$\overline{f_B}[(\mu_G - \mu_B)X] = \eta C + \overline{f_B}[(1-\eta)C + \alpha_I^* \mu_G X] \tag{3.9}$$

$$f_{B,I}^* = (1-\lambda) f_B + \lambda[1 - \gamma_I^*(1 - f_B)]\overline{f_B} \tag{3.10}$$

The logic is as follows. In current period t, $f_{G,t} = \overline{f_G}$, $f_{B,t} = \overline{f_B}$, $f_{BG,t} = \overline{f_{BG}}$. To induce the private agent not to offer bribe in period t and possibly go to LCSS-I, by inequality 3.1 we must have

$$\overline{f_B}[(\mu_G - \mu_B)X] \leq \eta C + \overline{f_B}[(1-\eta)C + \alpha_t \mu_G X]$$

From equation 3.9, we can solve α_I^*. Clearly, if $\alpha_t \geq \alpha_I^*$, above inequality will hold. Then in period $t+1$, the supervision effort goes back to the original level.To let a one time anti-corruption policy in some period t effective, the state of the economy in period $t+1$ must be in the Low corruption area I (L-I), i.e., $f_{B+1} \leq f_B^*$, where where f_B^* are the solutions of the equation 3.6.

By equation 3.3, in period $t + 1$

$$\begin{aligned} f_{B,t+1} &= \lambda(1-\gamma_t)\overline{f_B} + [\gamma_t\overline{f_B} + (1-\lambda)(1-\gamma_t\overline{f_B})]f_B \\ &= (1-\lambda)f_B + \lambda[1-\gamma_t(1-f_B)]\overline{f_B} \end{aligned}$$

So, we must have

$$f^*_{B,I} \geq (1-\lambda)f_B + \lambda[1-\gamma_t(1-f_B)]\overline{f_B}$$

From equation 3.10, we can solve γ^*_I. Clearly, if $\gamma_t \geq \gamma^*_I$, above inequality will hold.

Therefore, when the government sets a one time anti-corruption policy, not only does it have to increase the supervision effort on detecting the bribery behavior (α), but also it needs to consider the the supervision effort on detecting the behavior of intentionally placing obstacles (dereliction of duty) during the test (γ). Anti-corruption should work along both lines. ■

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