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Essays on Speculation

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by

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To María Elena and Mariana,

who accompanied me in this adventure.

Contents

List of Figures

List of Tables

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ABSTRACT OF THE DISSERTATION

Essays on Speculation

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Assets, as opposed to ordinary consumption goods or services, are long-lived and therefore may be traded repeatedly. The opportunity of retrading assets creates a new motive for trading based on the possibility of capital gains, namely, speculation.

In effect, some agents may participate as buyers not because they value the asset, but because they think that in the future it will be more valuable to somebody else. As long as storage costs —either financial or physical— are not too high relative to the anticipated price change, it may be rational for someone with that belief —however arrived at— to trade accordingly. This is particularly relevant for financial assets, characterized by low or null depreciation rates.

Two questions are raised by the existence of speculation:

1. How are prices affected? In particular, is it possible for asset prices to either temporarily or permanently differ from its fundamental value? Do asset prices "reflect" in some coherent way all information that is pertinent for estimating their fundamental value?

2. How is the resource allocation affected? In particular, does the volume of trade increase or is it the case that speculators' presence substitutes for regular investors? Will speculation promote a change in the market structure?

These essays discuss from a theoretical perspective two consequences of the existence of speculation in asset markets, namely, the possibility that informational efficiency does not obtain, and the bias towards intermediated market structures which the need to defend from speculators creates.

Chapter 2, entitled "Informational Efficiency and Competition", concludes that it is competition and not the existence of information itself what determines the informational content of prices. Chapter 3, "On the Limits to Speculation in Centralized versus Decentralized Market Regimes", argues that a market maker can mitigate the adverse selection costs that speculators impose on regular investors. Chapter 1 offers a survey of the microeconomic theory of speculation.

CHAPTER 1

Speculation in asset markets:

a survey

Abstract

This survey covers the microeconomic theory of speculation in asset markets, since the development of the economics of information. It starts with a description of Walrasian exchange economies, both in general equilibrium —the Arrow-Debreu model— and in partial equilibrium. Speculation, it is explained, is an incomplete-market phenomenon. It proceeds by analyzing more general voluntary trade environments, with a focus on whether or not differences in information are a valid source for belief heterogeneity. Necessary and sufficient conditions of the no-trade theorem are discussed. The review ends with a study of the bubble and speculative attack literatures, as the most prominent consequences of the existence of speculation.

CHAPTER 2

Informational Efficiency and Competition

Abstract

In a market with random pairwise meetings among differentially informed traders, it is shown that asset prices cannot reflect private information. When informed speculators are able to trade anonymously, standard signaling arguments guarantee that they will choose to pool themselves by making the same offer an uninformed investor would. This result is robust to the informational structure of the game, although the volume of trade might vary. However, when traders have to compete for the right to be matched, the economy becomes informationally efficient.

CHAPTER 3

On the limits to speculation

in centralized versus decentralized market regimes

Abstract

Speculation creates an adverse selection cost for utility traders, who will choose not to trade if this cost exceeds the benefits of using the asset market. However, if they do not participate, the market collapses, since private information alone is not sufficient to create a motive for trade. Therefore, there is a limit to the amount of speculative transactions that a given market can support. We compare this limit in decentralized versus centralized market regimes, finding that the centralized regime is more prone to speculation than the decentralized one: the transaction fees charged by an intermediary diminish the individual return to information, so that for a fixed value of trading, more speculative transactions can be supported. The analysis also suggests a reason for the existence of intermediaries in financial markets.

Chapter 1

Speculation in asset markets: a survey

1.1 Introduction

To speculate in asset markets is to trade motivated exclusively by the possibility of capital gains. It involves a belief, possibly divergent from the rest of market participants, that leads the speculator to bet against them. It also involves a disposition to act based on that belief. To explain the phenomenon of speculation, then, is to explain under what conditions we are likely to observe belief-based trading, and also how this trading activity changes market outcomes.

We start off by analyzing the conditions to observe speculation. These will be conditions on the market structure, and on the sources of belief heterogeneity. Section 1.2 reviews Walrasian exchange economies, the standard model for competitive environments, where it is established that a pre-condition for the existence of speculation is that the market structure be incomplete. Section 1.3 extends the analysis to more general voluntary trade arrangements, stressing the betting component of it, and inquires whether differences in information are a valid source of belief heterogeneity. Section 1.4 covers the consequences of speculation, especially the possibility of bubbles. Section 1.5 concludes.

The primary focus of this survey, it must be emphasized, is on the microeconomic theory of speculation. This means the discussion of the period preceding the development of the economics of information, with macroeconomic orientation —where the accent is placed on the issue of economic stability/instability— is left aside. We will only cite those results that are relevant to the more recent and formal discussion.

1.2 Walrasian exchange economies with heterogeneous beliefs

Walrasian theory of markets emphasizes price taking and market taking: all market participants (consumers and producers) regard prices and open markets as given, and behave rationally with respect to those parameters (Makowski and Ostroy, 1995). It is in this context that the present section addresses the problem of behavior under heterogeneous beliefs. To focus on informational issues, we deal only with an exchange economy.

Debreu (1959) first noticed that extending Walras' model of competitive markets to an intertemporal and uncertain environment could be done simply by refining the definition of commodity. Indeed, by treating a liter of milk today as a different object than a liter of milk tomorrow, and the liter of milk tomorrow if it happens to rain as differently as if it does not —that is, by enlarging the commodity space— one is able to analyze dynamic economies operating under uncertainty. There, he makes the fiction of contingent claims being available as well as open markets for all commodities just as in the certainty case. Thus, there is trading in $LT\Theta$ markets: L goods, T dates and Θ states. In this view, the demand for assets originates on the desire to move consumption across dates and states, that is, (financial) assets are useful because they allow consumption smoothing to risk-averse individuals.

1.2.1 The basic framework: the two-period complete-market model

In this section we briefly review the standard theory. Let $i \in I$ denote individuals, $\ell \in L$ (perishable) consumption goods, $t \in \{0, 1\}$ time periods, $\theta \in \Theta$ states of nature or descriptions of the world in $t = 1$, $\Theta_0 \equiv \Theta \cup \{0\}$, $\boldsymbol{\pi}^i$ prior beliefs over Θ, $w_t^i \in R^{L(\Theta+1)}$ endowments, $u_i(c_\theta^i) \in C^2$ utility functions over consequences, $k \in K$ assets, and q_k the price of an asset k.

Debreu (1959) considers the case where there is complete agreement with regard to the possible events (but not necessarily with respect to their likelihood) and where all information is public. Hence, individuals choose the consumption vector $\mathbf{x}^i$ of L commodities indexed by time t and states θ as to

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \widehat{\mathbf{p}}_0'(\mathbf{x}_0^i - \boldsymbol{\omega}_0^i) + \sum_{\theta \in \Theta} \widehat{\mathbf{p}}_\theta' \mathbf{d}_\theta^i \leq 0 \tag{1.1}$$

where $d_{\theta\ell}^i$ is the net purchase of claims to good ℓ in state θ, and thus $0 \leq x_{\theta\ell}^i \leq \omega_{\theta\ell}^i + d_{\theta\ell}^i$ $\forall\theta \in \Theta$, $\ell \in L$. $U^i(\mathbf{x}^i)$ is meant to be the von Neumann-Morgestern utility function $U^i(\mathbf{x}^i) = \sum_{\theta\in\Theta} \pi_\theta^i u(x_{\theta 1}^i, ..., x_{\theta L}^i)$. Thus, the budget constraint can be rewritten as $\widehat{\mathbf{p}}(\mathbf{x}^i - \mathbf{w}^i) \leq 0$, and the problem becomes

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \widehat{\mathbf{p}}'(\mathbf{x}^i - \mathbf{w}^i) \leq 0 \tag{1.2}$$

A striking fact is that this problem does not formally differ at all from the standard problem in consumer theory under certainty. This formulation stresses the fact that the consumer is choosing different bundles of consumption goods by buying a special kind of financial asset called "contingent claim".

The equilibrium is characterized by:

Definition 1 *A Walrasian equilibrium for the exchange economy* $\mathcal{E} = \{(\mathbf{w}^i, U^i)_{i\in I}\}$, *is a vector* $(\widehat{\mathbf{p}}, \mathbf{x})$ *such that:*

1. $\mathbf{x}^i \in \{\arg\max U^i(\mathbf{x}^i)$ *subject to* $\widehat{\mathbf{p}}'\mathbf{x}^i \leq \widehat{\mathbf{p}}'\mathbf{w}^i\}\ \forall i \in I$

2. $\sum_{i=1}^{I} x^i_{\theta\ell} = \sum_{i=1}^{I} w^i_{\theta\ell}\ \forall \theta \in \Theta_0, \ell \in L$

Theorem 1 *In this economy, the resulting allocation is Pareto-optimal ex-ante and ex-post irrespective of beliefs.*

This proposition, the first theorem of welfare economics, establishes that all gains from trade are exploited in one round of trade. Moreover, if markets were to reopen after the true state is known, no trade would occur since the allocation would still be Pareto-optimal (that is, also Pareto-optimal in an ex-post sense). To see this, one only needs to verify that the gradient vector of every individual's utility function is proportional to each other, both ex-ante and ex-post. Ex-ante, we have that $\nabla U^i(\mathbf{x}^i) \propto \nabla U^j(\mathbf{x}^j)$ because there is a market open for each argument of the utility function. Starting from the first order conditions of (1.2), we have

$$\left\{\nabla U^i(\mathbf{x}^i) = \lambda^i \widehat{\mathbf{p}} \text{ and } \nabla U^j(\mathbf{x}^j) = \lambda^j \widehat{\mathbf{p}}\right\} \Rightarrow \left\{\frac{\nabla U^i(\mathbf{x}^i)}{\lambda^i} = \widehat{\mathbf{p}} = \frac{\nabla U^j(\mathbf{x}^j)}{\lambda^j}\right\}. \tag{1.3}$$

Ex-post, when uncertainty is resolved, the gradient vectors are going to be modified by a proportional factor $\begin{cases} \frac{1}{\pi_\theta} & \text{if } \theta \text{ occurred} \\ 0 & \text{otherwise} \end{cases}$ by Bayes' rule. If we only consider the part of the vector $\mathbf{x}^i$ that contains consumption of goods in the state that actually materialized, $\mathbf{x}^i_\theta$, we have

$$\left\{\frac{1}{\pi^i_\theta}\nabla U^i(\mathbf{x}^i_\theta)=\frac{1}{\pi^i_\theta}\lambda^i\widehat{\mathbf{p}} \text{ and } \frac{1}{\pi^j_\theta}\nabla U^j(\mathbf{x}^j_\theta)=\frac{1}{\pi^j_\theta}\lambda^j\widehat{\mathbf{p}}\right\} \tag{1.4}$$

$$\Rightarrow \left\{\frac{\frac{1}{\pi^i_\theta}\nabla U^i(\mathbf{x}^i_\theta)}{\frac{\lambda^i}{\pi^i_\theta}}=\widehat{\mathbf{p}}=\frac{\frac{1}{\pi^j_\theta}\nabla U^j(\mathbf{x}^j_\theta)}{\frac{\lambda^j}{\pi^j_\theta}}\right\}.$$

Therefore, the opportunity of trading after uncertainty is resolved cannot improve welfare, and the re-opening of the market will not induce consumers to trade. What this implies, though, is that —in this economy— forecasting future asset prices is not an issue as there will not be open markets in the future, and —subsequently— there can be no speculation.

In effect, any differences in belief with respect to the likelihood of any particular state happening creates at time 0 a trade of contingent claims directly, that will pay consumption goods if the state materializes, so that reversing the position will not be necessary.

Arrow (1953) considers an otherwise similar economy, except that the securities that can be traded are not contingent claims but "pure securities" (also known as "Arrow securities"), that is, claims to 1 unit of account in state θ. There is trading in Θ state-claims before the resolution of uncertainty, and in L goods after it. This structure requires consumers to forecast consumption good prices $\mathbf{p}_\theta$ for each state. However, since the only uncertainty refers to the state that will materialize (endowments), it seems natural to assume that everyone agrees as to what prices will prevail *if* state θ occurs[1]. Yet, beliefs differ as to how likely it is that state θ occurs. Thus, each consumer solves

$$\max_{\{\mathbf{x}^i,\widehat{\mathbf{a}}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \mathbf{p}_0'\mathbf{x}_0^i + \widehat{\mathbf{q}}'\widehat{\mathbf{a}}^i \leq \mathbf{p}_0'\boldsymbol{\omega}_0^i \text{ and } \left\{\mathbf{p}_\theta'\mathbf{x}_\theta^i \leq \mathbf{p}_\theta'\mathbf{w}_\theta^i + \widehat{a}_\theta^i\right\}_{\theta\in\Theta} \tag{1.5}$$

where $\widehat{q}_\theta$ is the price today of a pure asset that pays off in state θ and $\widehat{a}_\theta^i$ is the net purchase by person i of an Arrow security that pays off in state θ. However, it is clear that $\mathbf{p}_\theta'\mathbf{x}_\theta^i = \mathbf{p}_\theta'\mathbf{w}_\theta + \widehat{a}_\theta$ if there is local non-satiation. Then, we have that (1.5) can be written as

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \mathbf{p}_0'\mathbf{x}_0^i + \sum_{\theta\in\Theta} \widehat{q}_\theta(\mathbf{p}_\theta'\mathbf{x}_\theta^i - \mathbf{p}_\theta'\mathbf{w}_\theta^i) \leq \mathbf{p}_0'\mathbf{w}_0^i \tag{1.6}$$

which is equivalent to the contingent claim case when $\widehat{p}_{s\ell} = \widehat{q}_s p_{s\ell}$. This means that the two economies are equivalent, in the sense that the consumption sets that these markets give rise to are the same. It follows that there is no special role for speculation either. Differences in beliefs as to the likelihood of a particular state happening explain trade, but there is no opportunity of capital gains because there are no price changes.

Finally, if instead of pure securities there were markets for ordinary securities, that is, promises of payment of variable numbers of units of account contingent on

[1] In Radner's (1972) terminology, expectations are "common". See section 1.2.3 below.

the occurrence of particular states, or put another way, bundles of pure securities, matters would not be different as long as we still have complete markets. In effect, if R is the $\Theta \times K$ matrix that contains as columns the state-contingent payoffs of the K assets in the Θ states, and if R is full rank (the complete-markets condition) then R^{-1} is the matrix specifying the portfolios of ordinary assets required to re-create a pure security for each state. Therefore, the consumer has the same options as before[2].

One should emphasize, however, that although there can be no speculation in this setting, asset markets do offer the opportunity of gambling, in the sense that people bet on the occurrence of particular states every time they choose to "put" a larger consumption bundle or larger units of account on them. Differences in opinion $\boldsymbol{\pi}^i$ give rise to trade, even if there were no other motives. Likewise, asset prices are affected by beliefs and their dispersion. Varian (1985) analyzes such relationship, concluding that dispersion of beliefs may lower or increase asset prices depending on how fast risk aversion responds to wealth.

1.2.2 More time periods and asymmetric information

The extension of the previous framework to more periods requires a reinterpretation of the concept of state and the modeling of uncertainty. The standard approach is to think of a state as a complete history, that is, a list of all actions taken by all

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[2] This implicitly assumes the possibility of unlimited short sales, for no restrictions are put on the sign of the entries in R^{-1}. We will come back to this point later.

individuals and nature at every time. Events, then, become the partial development of history through time.

It is possible that some events do not affect payoffs directly but may still affect beliefs (for instance, some moves from nature like sunspots). We will call them "pure informational events" to distinguish them from the "real events" which we have considered so far. Then, let us think of a state $\omega \in \Omega$ as being composed of two parts, $\omega = (\theta, \eta)$, where θ is a specification of the history of payoff-relevant actions whereas η is of payoff-irrelevant actions.

In what follows, it will be useful to recall a few definitions.

Definition 2 *A set* $H = \{h_n\}_{n=1}^{H}$ *is called a partition of a set* $\Omega = \{\omega_m\}_{m=1}^{\Omega}$ *iff* $\bigcup_{n=1}^{H} h_n = \Omega$ *and* $h \cap h' = \phi \ \forall h, h' \in H$.

Let us denote by $h(\omega)$ the element of H that contains ω.

Definition 3 *Let* H *and* H^* *be two partitions of* Ω. H *is said to be finer than* H^* *if* $h(\omega) \subseteq h^*(\omega)$ *for all* $\omega \in \Omega$. *If* H *is finer than* H^*, *then* H^* *is said to be coarser than* H.

Events, then, are the elements of partitions $h_t \in H_t$. The set of events will form a tree[3] represented by a sequence of partitions of Ω, $\{H_t\}_{t=0}^{T}$, where H_t is finer than H_{t-1} (reflecting the increased knowledge) and $H_T = \Theta$.

[3] Formally, a tree is a set of nodes such that each node has a unique predecessor. This is necessary to get a unique association between a state and a history, for if a terminal node had more than one backward path the knowledge of the terminal node would not suffice to uniquely pin down what actually happened.

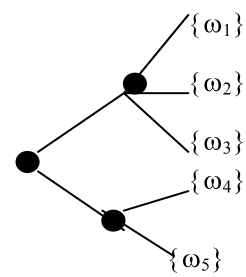

Figure 1-1: The event tree.

Figure 1-1 illustrates a situation where there are five states, each being a specification of, say, the actions of two persons at $t = 0$ and the choice of nature at $t = 1$. The states are given by $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, and the tree can be represented by the partition sequence $\{\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}, \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5\}\}, \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}\}$.

The list of actions will be enough to fully describe the world as long as everybody understands the causal connections with other variables. For instance, prices can be inferred unambiguously from quantities demanded and supplied when all individuals share the same model. We will assume this is indeed the case unless otherwise noted.

Contrary to what we have done so far, we will also want to explicitly consider the possibility that different people have different information, that is, that not all aspects of reality are revealed simultaneously to all individuals. If the definition of a state of the world is going to be fully comprehensive, that is, to specify all aspects of reality potentially pertinent to decision making, then it must include not only the set of actions taken but also who knows that those actions were taken, and who knows who is in possession of that knowledge, etc.

One way to achieve this is to define the informational structure as a state space Ω together with a collection of partitions of Ω, $\{H_t^i\}_{t=1,\ldots,T}^{i=1,\ldots I}$. This allows the definition of "personal" events as long as $h_t^i \neq h_t^j$ (consistent with the idea that what each person sees of a common reality is different, and therefore, with the existence of asymmetric information), while at the same time maintaining the identification of the state with all levels of mutual knowledge (if I know at time t that the state is ω, I also know what each person considers as possible $\{h_t^i(\omega)\}_{i=1}^I$, and what each of them thinks about what everybody else considers as possible, etc.[4]).

In cases where there is uncertainty but information is symmetric, we drop the superscript i since $\{H_t^i\}_{t=0}^T$ is common to all consumers. Similarly, we drop the time subscript when there is only one period left.

As for the previous models, with symmetric information, contingent claims are now related to events and not to states, and the same holds for contingent payments of assets, either pure or ordinary.

As for beliefs, rationality requires them to be updated according to Bayes' rule,

[4] Aumann suggests this is tautological, for part of the definition of the state is what each agent knows abouth the knowledge of others. Nevertheless, the issue is not completely resolved. See Geanakoplos (1993), the literature on hierarchies of beliefs, and the discussion in Gul (1998) and Aumann's reply (1998).

that is,

$$\pi^i_{h_{t+1}} = \begin{cases} \dfrac{\sum_{\theta \in h_{t+1}} \pi^i_\theta}{\sum_{\theta \in h_t} \pi^i_\theta} & \text{if } h^i_{t+1} \subseteq h^i_t \\ 0 & \text{otherwise} \end{cases} . \tag{1.7}$$

1.2.3 Incomplete markets

Radner (1972) presents what is nowadays the standard framework to analyze incomplete-market symmetric-information economies, that is, cases where "at every date and for every commodity there will be some future dates and some events at those dates for which it will not be possible to make current contracts for future delivery contingent on those events". Expectations are *common* if all traders associate the same prices to the same events, and are *consistent* if the anticipated excess demands are zero. It should be noted that we have already assumed common and consistent expectations throughout. In this setting, there is a nontrivial role for sequential trading because the opportunity of retrading the same assets is valuable as it enlarges the budget set. Hence, retrading acts as a substitute for inexistent markets[5].

Hirshleifer (1975) exemplifies this idea. His model corresponds to a two-good economy in which there are two rounds of trade, before and after the arrival of (private[6]) information, and consumption occurs at the last date. However, markets are

[5] This idea is further developed in Duffie and Huang (1985).

[6] Although the text considers the possibility that the information received by each trader is different, the model makes no special treatment of it. Moreover, since priors differ, posteriors would differ too, even though the information received is the same. See section 1.2.

incomplete since there are contingent claims for only one good; the second good must be traded *incontingently*. In the anticipation of a price change, individuals would trade to move from their endowment to a trading position, while they would trade again to go to the consumption position once the uncertainty is resolved. The incompleteness creates the need for trading in the second round, for as we have seen, under a complete market regime consumers could choose directly the final consumption bundle of contingent commodities, and any differences in beliefs would be reflected on date-zero prices. In fact, as pointed out in Hirshleifer (1977), "in the prior round each trader would be able to buy a portfolio covering his desired consumption baskets in the light of the alternative possible information-events as well as over the different state-contingencies".

Thus, the intuitive conclusion that speculation occurs owing to differing anticipations of price changes (and cannot be a consequence of a redistribution of risks) holds because price changes are a necessary condition for completing the market via contingent trading, i.e., to substitute for missing markets. Rubinstein (1975) expands on this idea by considering a three-date Arrow-Debreu economy, where a complete set of real-event contingent claims is available at every date but no information-event contingent claim and there is only one consumption good. Apart from showing the Pareto-optimality of this economy, which holds because retrading substitutes for market-completeness, he proposes three alternative ways to think of the concept of prices reflecting all available information: *non-speculative beliefs* are those beliefs for which no portfolio revision is an optimal strategy; *consensus beliefs* are those beliefs which if held by all individuals in an otherwise similar economy, would generate the same equilibrium prices as in the actual heterogeneous economy.

Grundy and McNichols (1990) show that if it is known in advance that the market will be open in the future, it is not clear at all that agents would prefer *a priori* to trade in any of the available rounds in particular. In other words, if the market is known to reopen, it might be very active, but it is only so because it is necessary to complete transactions that could have been done in the first round but just weren't. Therefore, giving a nontrivial role to future rounds of trade requires the incompleteness of the set of available markets.

Up to this point, we have seen that existing asset markets allow individuals to smooth consumption and/or gamble. Speculation —trading and retrading based on belief heterogeneity— arises only if the market structure is incomplete, and has to be seen as the natural consequence of the need to substitute for missing markets. The next section generalizes the economy to consider any voluntary trading arrangement, that is, markets where any trade must be acceptable to all participants —be it under given prices or not—. In this setting, the question of whether or not differences in information are a valid source of belief heterogeneity is explored.

1.3 The possibility of speculation

1.3.1 Speculation as a betting game

Ultimately, when an income stream is chosen over another, one could say that there is an implicit bet over the likelihood of the states where income is increased. In fact, buying a share in the hope of a price rise is to bet on a price increase. More precisely,

Definition 4 *A bet is a function $b : \Theta \longrightarrow \mathbb{R}^I$ specifying for each state θ a vector of monetary payoffs $\mathbf{b}_\theta = (b^1_\theta, ..., b^I_\theta) \neq \mathbf{0}$ such that, $\forall \theta \in \Theta$, $\sum_{i \in I} b^i_\theta = 0$. Player i is said to have bet \$$b$ on state θ if, whenever some state $\theta' \in \Theta \diagdown \{\theta\}$ occurs, $b^i_{\theta'} = -b$.*

Thus, a bet is a vector of transfers that must be agreed upon by all participants.

In the Walrasian model, it is possible to define the indirect utility function of consumption by

$$\begin{aligned} v^i(\mathbf{b}^i) &= v^i(b^i_0, b^i_1, ..., b^i_\Theta) = \max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to} \\ & \left\{\mathbf{p}'_\theta \mathbf{x}^i_\theta \leq \mathbf{p}'_\theta \mathbf{w}^i_\theta + b^i_\theta\right\}_{\theta \in \Theta_0} \end{aligned} \tag{1.8}$$

It is important to kept in mind, though, that implicit on it there are beliefs, and there is knowledge that provides a basis for them. The function $v^i(\mathbf{b}^i)$, then, evaluates income streams on the basis of preferences, beliefs, and endowments. In particular, $v^i(\mathbf{b}^i)$ can evaluate bets.

A bet $\mathbf{b} = (\mathbf{b}^1, \mathbf{b}^2, ..., \mathbf{b}^I)$ will be carried over only if all individuals agree to it, that is,

$$v^i(\mathbf{b}^i) \geq v^i(\mathbf{0}^i) \ \forall i \in I \tag{1.9}$$

One may wish to separate out what part of a trade is due to differences in beliefs and what part is due to consumption smoothing. Let $\widetilde{\mathbf{b}}$ be the bet that would be carried over if all beliefs would coincide[7], that is, $\boldsymbol{\pi}^i = \widetilde{\boldsymbol{\pi}} \ \forall i \in I$. Such a state-contingent transfer would of course be justified by the structure of endowments and risk aversion. Then, $\widehat{\mathbf{b}} \equiv (\mathbf{b} - \widetilde{\mathbf{b}})$ would be due entirely to differences in beliefs. Speculation, in this sense, would be the act of betting on some states based on deviant expectations. For if beliefs would coincide, then there would be no speculation at all.

It can readily be seen that this is exactly what is done in a Walrasian economy where Arrow–securities are traded. Security markets give the opportunity of betting in the above sense, because of differences in beliefs, endowments, or risk aversion. The only special feature of Walrasian economies is the way in which the available bets are determined, given by $\left[b_0^i = -\sum_{k=1}^{K} q_k a_k \text{ and } b_\theta^i = \sum_{k=1}^{K} r_{\theta k} a_k \right]$ and the fact that $\mathbf{q}$ satisfies market-clearing.

[7] We simply assume such a bet exists.

The present analysis, then, has the advantage that it includes, but is not limited to, Walrasian economies, for it can serve as a characterization of any voluntary process of trade. This is the environment used by Milgrom and Stokey (1982) to study the rational expectations equilibria, to which we turn in the next section.

1.3.2 Betting is ruled out by common knowledge

So far we have assumed belief heterogeneity without paying special attention to the sources of that heterogeneity. Indeed, the Walrasian tradition has been to treat beliefs as exogenous variables, just like utility functions. Each person then makes decisions based on observed prices without worrying about the origin of those prices because the information concerning why a relative price took a particular value — whether it can be traced back to beliefs, preferences, technology, etc.— is completely useless. An implicit assumption, then, is that information is symmetric, that is, the event tree $\{H_t\}_{t=1}^{T}$ is common.

However, when beliefs are affected by information, and information is not public but heterogeneously distributed among society members —a more realistic setting— any inference that one can obtain by observing other agents' behavior becomes valuable because it may improve decision making.

Common wisdom points to differences in information as the main source of belief heterogeneity. Speculators, in possession of more or better information, would be better-than-average forecasters earning a return for their social contribution in keeping prices in line with available information, which improves the quality of investments. This is in gross terms the view of Working (1953), and also what Fama (1970) had in mind when discussing the efficient market hypothesis.

Lintner (1969) illustrates how prices would aggregate these disperse judgments, in a partial equilibrium model in the Walrasian tradition. In the particular case of normally distributed returns and exponential utility functions —the "normal-exponential model", which later became standard in finance theory— it is shown that the equilibrium asset price is measurable with respect to the vector of private signals $\{h^i\}_{i=1}^{I}$.

This view, however, was challenged by the then novel theory of rational expectations, which basically asserts that if agents are rational, they should recognize that the way prices are formed makes them useful pieces of information in their own right. Moreover, under some conditions, they become sufficient statistics of all private signals, providing a better guide than each particular piece of information by itself. In the normal-exponential setting, Grossman (1976) shows that private information is redundant once the price is known. But this entails a paradox: "When a price system is a perfect aggregator of information it removes private incentives to collect information"; but then, if no information is collected, there is no information to be transmitted by prices. In turn, the absence of information in prices generates returns to be appropriated by those who gather information.

Grossman and Stiglitz (1980) treated this issue by introducing the notion of noise traders. In their model, a random shock to the asset supply is added so that no individual can perfectly infer the signal vector just by looking at the price. As the inferential process is limited in this fashion, private returns to information gathering activities are restored. This class of model is often referred to as noisy rational expectations models, which were developed further by Hellwig (1980), Diamond and Verrechia (1981), Admati (1985), Kyle (1985), and Admati and Pfleiderer (1986).

Although the noise trader approach proved useful for analyzing information acquisition and aggregation under different market structures, the exogenous behavior of some agents on which it relies is not entirely satisfactory, for it raises the doubt as to what is the necessary feature to obtain incomplete inference, so that a competitive equilibrium can exist, be it some behavioral irrationality, limited inference capabilities, or bounded rationality of some other kind. In other words, the source of noise trading is at least as obscure as the source of the differences in beliefs that these models tried to illuminate.

Fortunately, some striking results in the literature on common knowledge, started by Lewis (1969) in philosophy and Aumann (1976) in economics, helped clarify the issue of whether or not differences in information are a valid source of belief heterogeneity. Although perhaps creating some extra confusion initially, with answers overly stated on the negative side in the pioneering work of Milgrom and Stokey (1982), it was possible to understand that full revelation is obtained with the conjugation of common knowledge of actions, common priors and an initial Pareto-optimal allocation. It is important to stress the fact that none of these ingredients is traceable directly to the individuals' own rationality. Therefore, the no-speculation (and/or no-trade) paradox was not due to the rationality assumption[8] but to the conjunction of ancillary assumptions of the rational expectation hypothesis, like common priors. The remainder of this section will be devoted to review these results in more detail.

Subsection 1.2.2 showed that information can be modelled as the knowledge of an event $h_t^i(\omega)$, which is a particular element of a partition H_t^i of Ω. This has the interpretation that agent i knows at time t that the true state is one of the elements of $h_t^i(\omega)$, among which he cannot recognize it —reflecting the remaining uncertainty— and that he also can safely discard any $\omega' \in \Omega \backslash h_t^i(\omega)$. What follows is an analysis of a static problem and consequently the time subscript is dropped.

Since all the individuals' partitions $\{H^j\}_{j \in I}$ are known, then knowledge of $h_t^i(\omega)$ also implies knowledge of what other agents may know, for individual i cannot reject the possibility that individual j knows $h^j(\omega')$ for any $\omega' \in h^i(\omega)$. Mutual knowledge is, then, implied this way by the state and the informational structure.

Define the knowledge operator as

$$K_i(E) = \{\omega \in \Omega : h^i(\omega) \subseteq E\} \tag{1.10}$$

which has the interpretation that player i knows that event E occurred if E cannot be ruled out in any of the states he considers as possible. Similarly, when individual

[8] Of course, irrationality can eliminate the paradox too, but it is not necessary.

i at ω cannot reject any of the states in which j knows E, then i knows that j knows E:

$$\omega \in K_i \left(K_j(E)\right) \tag{1.11}$$

Lengthier iterations of the knowledge operator reflect higher levels of mutual knowledge. It will be useful now to bear in mind the following definitions:

Definition 5 *Let $\mathcal{H} = \{H_i\}_{i \in I}$ be a collection of partitions of a set Ω. The meet of $\mathcal{H}$ is its finest common coarsening, $\mathcal{M}(\omega) = \cup_I \cup_{i \in I} h_i(\omega)$, and the join of $\mathcal{H}$ is its coarsest common refinement, $\mathcal{J}(\omega) = \cap_{i \in I} h_i(\omega)$.*[9]

The join has a ready interpretation: it indicates what is the maximum knowledge in society, which would be obtained if everyone were to pool his information with everybody else's. It is clear that if information were public, everybody's partitions would coincide and be equal to their join. On the other hand, one can define information that is public in some states but not necessarily at every state: information is public at ω if $h_i(\omega) = h_j(\omega)\ \forall i, j \in I$. The meet, in turn, is useful to define what events are commonly known:

Definition 6 *(Aumann, 1976) An event E is common knowledge if $\mathcal{M}(\omega) \subseteq E$.*

Clearly, if $h_i(\omega) \subseteq E$ then agent i considers E to be true in every state he sees as possible. This is also true for every $i \in I$, since $h_i(\omega) \subseteq \mathcal{M}(\omega)$. Moreover, all

[9] *The meet and the join are themselves partitions of Ω, and they are unique. See Genakoplos (1993) for a proof.*

iterations of the form $K_k(K_j(...K_i(\omega)))$ are also true, so mutual knowledge is true even in infinite regressions.

Aumann not only develops this formal definition of common knowledge, but also proves the impossibility of agreeing to disagree: two individuals with common priors that obtain different information, cannot have differente posteriors if they are commonly known. This result forms the basis for the no-speculation theorems and the stronger no-speculation results.

Milgrom and Stokey (1982) provide the following[10]:

Theorem 2 *Suppose all traders are risk-averse, that the initial allocation is Pareto-optimal, that agents' prior beliefs are common, and that each player i observes the information conveyed by the partition H^i. If it is common knowledge at ω that* **b** *is a feasible trade and that each trader weakly prefers it to the zero trade, then every agent is indifferent between* **b** *and the zero trade. If all agents are strictly risk averse then* **b** *is the zero trade.*

Thus, Geanakoplos (1993), in an excellent survey on common knowledge, asserts that "The main conclusion is that an apparently innocuous assumption of common knowledge rules out speculation, betting, and agreeing to disagree". Specifically, he provides a proof of the following:

[10] Aside from notational differences, their theorem was stated with a softer assumption, namely, concordant beliefs rather than common priors. See Morris (1994).

Theorem 3 *Let* $(\Omega, (H_i, A_i, s_i)_{i\in I})$ *be given, where* Ω *is a set of states of the world,* H_i *is a partition on* Ω, A_i *is an action set, and the strategy* $s_i : \Omega \longrightarrow A_i$ *specifies the action agent* i *takes at each* $\omega \in \Omega$, *for all* $i \in I$. *Suppose that* s_i *is generated by the decision rule* $\psi_i : 2^\Omega \longrightarrow A_i$ *satisfying the sure-thing principle*[11]. *(Thus* $s_i(\omega) = \psi_i(h_i(\omega))$ *for all* $\omega \in \Omega$, $i \in I$.*) If for each* i *it is common knowledge at* ω *that* s_i *takes on the value* a_i, *then there is some single event* E *such that* $\psi_i(E) = a_i$ *for every* $i \in I$.

In the context of this survey, the ψ_i function corresponds to the strategy that player i follows (indicating what to do at every information set $h_i(\omega)$ he may find himself in). The theorem, then, establishes that the same action profile could have been obtained with symmetric information in an otherwise similar game. Hence, the informational asymmetry is not the explanation for the observed actions.

Special cases of the above theorem are Aumann's agreeing to disagree result and Milgrom and Stokey's no-trade theorem. In fact, the following example is provided by the latter to see the role of common knowledge in the no-trade theorem:

Example 1 *(Milgrom and Stokey 1982) There are two payoff-relevant states,* $\Theta = \{\theta_1, \theta_2\}$. *Two players must simultaneously decide whether they accept or reject a bet in the following terms: if state* θ_1 *materializes, player 2 (she) pays \$1 to player 1 (he); if* θ_2 *occurs, the reverse payment is carried out. Before making a decision, however,*

[11] *The sure-thing principle says that if* $\psi(A) = \psi(B) = a$ *and* $A \cap B = \phi$, *then* $\psi(A \cup B) = a$. *See Savage (1972).*

each of them gets to see a private signal (information event) within the following sets: $H_1 = \{\{\eta_1, \eta_2\}, \{\eta_3, \eta_4\}, \{\eta_5\}\}$ *and* $H_2 = \{\{\eta_1\}, \{\eta_2, \eta_3\}, \{\eta_4, \eta_5\}\}$*, which in fact are two distinct partitions of* $\Lambda = \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5\}$*. They have common priors on* $(\Theta \times \Lambda)$ *given by*

	η_1	η_2	η_3	η_4	η_5
θ_1	*0.20*	*0.05*	*0.05*	*0.15*	*0.05*
θ_2	*0.05*	*0.15*	*0.05*	*0.05*	*0.20*

Suppose the true message is η_3*. Should they bet? The answer is no if there is common knowledge of rationality. To see this, imagine first that the players are rational but they are not aware of their opponent's rationality. Then, each player computes the expected value of betting according to*

$$\begin{aligned} E[u_1|\{\eta_3, \eta_4\}] &= \frac{2}{3}(1) + \frac{1}{3}(-1) = \frac{1}{3} > 0 \text{ and} \\ E[u_2|\{\eta_2, \eta_3\}] &= \frac{1}{3}(-1) + \frac{2}{3}(1) = \frac{1}{3} > 0. \end{aligned}$$

As both are positive, they would accept. Moreover, if each of them knew that his/her opponent is rational too, they would not only check their own answer but also their opponent's answer, according to the information they could have received. Indeed, player 1 knows that his opponent could have received either $\{\eta_2, \eta_3\}$ *or* $\{\eta_4, \eta_5\}$*, but* *as*

$$\begin{aligned} E[u_2|\{\eta_2,\eta_3\}] &= \frac{1}{3}(-1)+\frac{2}{3}(1)=\frac{1}{3}>0 \text{ and} \\ E[u_2|\{\eta_4,\eta_5\}] &= \frac{20}{45}(-1)+\frac{25}{45}(1)=\frac{1}{9}>0, \end{aligned}$$

her acceptance does not tell him anything new and his original calculation is still the most accurate. Similarly, she observes that his behavior would be the same irrespective of having received messages $\{\eta_1,\eta_2\}$ *or* $\{\eta_3,\eta_4\}$*, since*

$$\begin{aligned} E[u_1|\{\eta_1,\eta_2\}] &= \frac{25}{45}(1)+\frac{20}{45}(-1)=\frac{1}{9}>0 \text{ and} \\ E[u_1|\{\eta_3,\eta_4\}] &= \frac{20}{30}(1)+\frac{10}{30}(-1)=\frac{1}{9}>0. \end{aligned}$$

However, the analysis would be different with one more level of knowledge of mutual rationality. Suppose that not only player 1 knows she is rational, but also that he knows that she knows that he is rational. In that case, when receiving message $\{\eta_3,\eta_4\}$ *he knows she could have received messages* $\{\eta_2,\eta_3\}$ *or* $\{\eta_4,\eta_5\}$*, which in turn implies she would consider cases where he receives messages* $\{\eta_1,\eta_2\},\{\eta_3,\eta_4\}$*, or* $\{\eta_5\}$*. Then she knows, he reasons, that if he accepts she can safely assume the message was not* $\{\eta_5\}$*, since*

$$E[u_1|\{\eta_5\}]=\frac{5}{25}(1)+\frac{20}{25}(-1)=-\frac{3}{5}<0$$

But then he must conclude that she will reject the bet if she receives the message

$\{\eta_4, \eta_5\}$, *since*

$$E[u_2|\{\eta_4, \eta_5\}\backslash\{\eta_5\}] = \frac{15}{20}(-1) + \frac{5}{20}(1) = -\frac{1}{2} < 0.$$

Similarly, as she would reject the bet when the message is $\{\eta_1\}$ *and he knows it, he in turn would not accept when receiving* $\{\eta_1, \eta_2\}$ *as a message. But this leaves us with the message* η_3 *as the only candidate for simultaneous acceptance of the bet. As in fact they both know it, in this case we have*

$$E[u_1|\{\eta_3\}] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0 = E[u_2|\{\eta_3\}]$$

where their expectations have converged (negating asymmetric information) and there are no gains from trade.

Remark 1 *All this complicated string of reasoning is embedded in the definition of Nash equilibrium. Once common priors on the possible states of the world are assumed, the assumptions of common knowledge of rationality and common priors on the opponent's behavior are invoked when looking at a Nash equilibrium. In effect, as shown in Aumann (1987), those assumptions are equivalent to correlated equilibrium, of which Nash equilibrium is a special case.*

The above example, for instance, has an associated normal form

$s_1 \setminus s_2$	*000*	*001*	*010*	*011*	*100*	*101*	*110*	*111*
000	***0,0***	*0,0*	***0,0***	*0,0*	*0,0*	*0,0*	*0,0*	*0,0*
001	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*
010	***0,0***	*.10,-.10*	***0,0***	*.10,-.10*	*0,0*	*.10,-.10*	*0,0*	*.10,-.10*
011	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*
100	*0,0*	*0,0*	*-.10,.10*	*-.10,.10*	*.15,-.15*	*.15,-.15*	*.05,-.05*	*.05,-.05*
101	*0,0*	*-.15,.15*	*-.10,.10*	*-.25,.25*	*.15,-.15*	*0,0*	*.05,-.05*	*-.10,.10*
110	*0,0*	*.10,-.10*	*-.10,.10*	*0,0*	*.15,-.15*	*.25,-.25*	*.05,-.05*	*.15,-.15*
111	*0,0*	*-.05,.05*	*-.10,.10*	*-.15,.15*	*.15,-.15*	*.10,-.10*	*.10,-.10*	*0,0*

where acceptance is represented by a 1 and rejection by a 0. The knowledge of the opponent's partition is implied by the knowledge of his/her strategy. This game has four pure-strategy Nash equilibria (in bold), none of them corresponding to the common wisdom that an auspicious private message will induce betting (strategy profile $\mathbf{s} = (110, 011)$ *in the example). Following theorem 10,* $\psi_1(\{\eta_3, \eta_4\}) = \psi_2(\{\eta_2, \eta_3\}) = \psi_1(\{\eta_3\}) = \psi_2(\{\eta_3\}) = 0$.

These findings explain Kreps' (1977) and Tirole's (1982) impossibility of speculation in rational expectations economies. The common prior and common knowledge assumptions are embedded on (most) rational expectations models[12], which explains Tirole's claim that "speculation relies on inconsistent plans and is ruled out by rational expectations". Moreover, rational expectations models also assume von Neumann-Morgestern utility functions, that is, the state preference approach. We discuss next the robustness of the no-trade result to each assumption separately:

Common knowledge

Even though breaking down common knowledge leads to trade, as we saw in example 1, Fudenberg and Levine (1994) show that solution concepts not involving common knowledge, like $\varepsilon-$self-confirming equilibria and $\varepsilon-$marginal best response distributions, also give rise to the same no-trade result as the asymptotic outcome of a learning process, where everything else is as above (common priors and ex-ante Pareto-optimal allocation). The reason for this is that any probability distribution over socially feasible outcomes that Pareto-dominates the endowment must involve no-trade. This means that the zero-sum feature of pure speculation is too strong.

Partitional information

Another extension that has been pursued is to adapt a less restrictive model of knowledge. In particular, it has been shown (see Samet (1990) and Geanakoplos (1989))

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[12] There is more than one definition of rational expectations equilibria. See McAllister (1990) for a discussion of different concepts, and a definition that explicitly considers beliefs about beliefs and common knowledge of rationality and market clearing as well. An interesting feature of his concept is that it does not involve full revelation.

that to represent an information structure by a partition of a state space is equivalent to assume not only that at each state the individual knows what he knows and knows that he knows it, but also that he knows that he does not know what he does not know. This latter assumption has been judged too demanding, particularly when facing very large state spaces. However, dropping this assumption does not automatically generate trade, for there is a minimum degree of irrationality needed.

Awareness and non-expected utility

Ghirardato (1995) provides a model of decision making in the spirit of Savage's (1972), except that the decision maker does not fully understand the connection between action and consequences, and is aware of that unawareness. An axiom about ignorance resolution allows him to obtain a representation of preferences over acts, which corresponds to an expected utility where the expectation is taken with respect to a non-additive belief function, a so-called non-expected utility; this is a formulation previously obtained under different axioms in the literature on "Knightian uncertainty" (see, for instance, Schmeidler(1989), and references therein; further support for non-expected utility but this time by a biological argument of fittest rather than axiomatically, can be found in Robson (1996)). An important result in this approach is in Dow and Werlang (1992), who assert that non-expected utility is a source of inaction, creating, for example, a "bid-ask spread property" for security trading, that is, a price-interval where the individual will not want to either buy or sell. Trade could occur after the arrival of information even with an ex-ante Pareto-optimal allocation, Ghirardato argues, because information may shrink this inaction interval. Just like in the case of non-partitional information structures, however, the extent to which trade is driven by irrationality or absence-of-omniscience is still under discussion.

Dow, Madrigal and Werlang (1990), on the other hand, claim that provided common knowledge of the desirability of the trade and state-additivity of preferences are assumed, the no-trade result goes through. In other words, it cannot be a consequence of learning from opponents' behavior (ex-post partitions need not be finer than ex-ante partitions) and, in consequence, an adverse selection phenomenon. Furthermore, as suggested in the previous paragraph, non-expected utility creates trade because of a lack of state-additivity.

Common priors

If priors were heterogeneous, even the same information would generate trade, for individuals would obtain different conclusions from it and could "agree to disagree"[13,14]. Morris (1995) challenges the so-called "Harsanyi doctrine", namely that differences in belief can only come from differences in information and that, therefore, the common prior assumption is not only reasonable but tautological. He argues that the basis for

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[13] Provided, of course, markets are incomplete. This scenario actually corresponds to the model in section 2.2.

[14] Not any difference in prior beliefs, however, will invalidate the no-trade result. See Morris (1994).

this claim is weak, for logic can only tell us how to change beliefs, not how to choose a prior. Moreover, the construction of the infinite hierarchy of beliefs that exhausts the uncertainty of each individual about others lends no support to the common prior assumption. Thus, the existence of common priors is not tautological, and there are good reasons to accept the subjective probability approach[15].

Ex-ante Pareto-optimality

Even holding to the Harsanyi doctrine, we could think of the no-trade results as establishing the impossibility of *purely* speculative markets, that is, as saying that private information cannot be a unique source of trading. Nevertheless, one can hardly say that at any point in time it is commonly known that there are only speculators in the market. A prior round of trade, for instance, may fail to produce a Pareto-optimal allocation, even with complete markets. As Grundy and McNichols (1990) show, if the market is known to reopen, individuals may well use both rounds to achieve their optimal consumption bundles.

The relevant question seems to be not whether private information in isolation can generate trade, but whether it is capable of affecting the total volume of trade in an economy where consumption smoothing, insurance, liquidity and any other possible motive already played a role. The answer to this question appears to be, at least at an intuitive level, a strong "yes". Ross (1989), for instance, says that "It is difficult

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[15] See footnote 2 above.

to imagine that the volume of trade in security markets has very much to do with the modest amount of trading required to accomplish the continual and gradual portfolio balancing inherent in our current intertemporal models."

From that perspective, models that explain the effects of speculation on prices and trading volume ought to include traders with diverse motives. The next section focuses on such models, with emphasis on price effects.

1.4 Consequences of speculation

1.4.1 Bubbles and crashes

It is said that a bubble exists on an asset if its price differs from its fundamental value, that is, the expected present discounted value of dividends. Many pieces of evidence prompt us to believe in the existence of bubbles. Perhaps the most compelling argument is that the observed market booms and sudden crashes observed throughout the history of financial systems are difficult to reconcile with a fundamentalist explanation, especially since it has been hard to find an event important enough to explain the magnitude of the changes. Classical examples are the October 1929 and October 1987 episodes, the South Sea Bubble and the tulip mania last century, documented in Kindleberger (1989) and White (1990) and discussed in Garber (1990). On the other hand, the excess volatility puzzle (Shiller (1981), Shiller (1990), discussed in Kleindon (1986)), namely, the cumulated evidence that asset prices move far in excess of what would be justified by changes in dividends, arises as indirect evidence in favor of non-fundamental determinants of asset prices.

The common prior approach

Tirole (1982, 1985) explains that bubbles cannot occur in infinite perfectly competitive economies under rational expectations, but they can in growing overlapping generations economies. To understand this, one has to see that an arbitrage argument breaks them down. If a bubble is known to be present in an asset, anybody could short-sell that asset to buy it after the burst. On the other hand, this argument cannot work in overlapping generations because the bubble may exceed an individual investors' life. Thus, if everyone believes that the price of an asset is going to rise indefinitely in the future, every new generation of investors will be willing to buy it in the certainty (or almost certainty with stochastic bubbles) of being able to sell it later. If the economy is growing, there is no potential problem of the price becoming higher than total wealth, which otherwise would rule out the bubble by a transversality condition.

In Jackson (1994), a set of private payoff-irrelevant signals may help to correlate bidding behavior (the market at each period is a Vickrey auction) and therefore to create and sustain a bubble. Since private signals are correlated, a good signal today is an indication of a high price tomorrow. Not using this information is harmful, provided everybody else is using it. Moreover, this equilibrium is strict, so it survives refinements.

Similarly, Bhattacharyya and Lipmann (1995), using a bargaining game, construct an equilibrium where the bubble builds up every period because there is uncertainty about individual wealth levels. Once the maximum wealth is reached, the bubble bursts. Their model departs from Tirole's in that there is bargaining rather than price-taking, and the equilibrium requires interim rater than ex-ante rationality (on this latter point see also Morris (1994)).

Lee (1995), taking a different approach but still based on rationality, explains market crashes as a failure in information aggregation. Following the cascade literature, he notes that traders may not use their information, say bad news, if before them many traders evidence good news. Indeed, it would be rational for them to "follow the crowd" if their information is more powerful than his own, or do nothing in the presence of transaction costs. But this will be true for any trader after him, because at that point learning stops. Later, any small change may trigger an abrupt change in behavior, since the true information vector will be revealed. This argument implies that crashes and avalanches are correcting mechanisms of previous bubbles (in the sense that total information in society warrants a different fundamental).

Hart and Tauman (1997) also have argued that market crashes may come as the outcome of endogenous information processing in the spirit of example 1 above, that is, as a getting-to-common-knowledge problem. Still, their conclusions are puzzling since they would seem to contradict the no-trade theorem.

Finally, Madrigal and Sheikman (1997) present a model in which a market maker sets prices strategically in order to profit from the information obtained from the order flow. They show that the price-functions may present discontinuities, which they interpret as crashes.

Heterogeneous prior models

Harrison and Kreps (1978) define speculation somewhat differently: "investors exhibit *speculative behavior* if the right to resell a stock makes them more willing to pay more for it than they would pay if obliged to hold it forever". They develop a partial equilibrium model with risk-neutral investors that hold heterogeneous expectations over the dividend process a single security follows. At every point in time, the equilibrium price must be (weakly) larger than what any particular investor might think the worth is —the difference being the bubble. It is important to note, though, that this concept actually corresponds to the option value associated to the possibility of resale, that is, to the liquidity of the asset. What heterogeneous expectations do is increase the value of the resale option. Stout (1995) adds that heterogeneity in posteriors is more likely to appear as a result of exogenous technological or economic developments, which are hard to interpret.

These ideas agree with Keynes' notion of long-term expectations, and how liquidity may affect the market outcome:

> "Of the maxims of orthodox finance none, surely, is more anti-social than

the fetish of liquidity, the doctrine that it is a positive virtue on the part of investment institutions to concentrate their resources upon the holding of 'liquid' securities. It forgets that there is no such thing as liquidity of investment for the community as a whole. The social object of skilled investment should be to defeat the dark forces of time and ignorance which envelop our future. The actual, private object of the most skilled investment to-day is 'to beat the gun', as the Americans so well express it, to outwit the crowd, and to pass the bad, or depreciating, half-crown to the other fellow." (The General Theory, page 155).

Along the same line, Harris and Raviv (1993) show that the heterogeneous prior assumption[16] generates predictions consistent with the data, which would be difficult to reconcile with the rational expectations approach. For instance, the fact that trade volume is greater after public announcements, and the positive correlation between volume and absolute price changes.

Detemple and Murthy (1994) analyze the consequences of prior heterogeneity in a continuous-time single-good economy. All investors observe the same news and evolution of random shocks to aggregate technology; yet, each type of agent holds different beliefs as to the prospects of investments —for arbitrary long periods

[16]When they say they assume common priors in the paper, they refer to the marginal over payoff-relevant events θ and not to the actual prior over states ω. As the conjugate posteriors are different ("what they call different models"), then their priors over Ω must differ.

of time— which leads them to hold different portfolios. Asset prices are weighted averages of those that would prevail in homogeneous agent economies. They do not, however, explicitly consider the issues of bubbles and crashes.

Daniel et al. (1998), in a provoking article, base the heterogeneity on well-established deviations from rationality[17] in the psychology literature, namely, investor over-reaction and biased self-attribution. The main argument starts with the observation that a large amount of evidence has accumulated that is inconsistent with the rationality assumption, most notably the excess volatility puzzle, the equity premium puzzle and a series of asset pricing anomalies. When the cited anomalies are incorporated, the argument goes, it is possible to reproduce parsimoniously the patterns in the data in a way that other forms of irrationality would not. A related study by Levine (1982) shows, by the same token, that some observed patterns in prices can be reproduced by a model that assumes some investors are fundamentalists —demand is the result of value assessment— while some are chartists —who believe changes in asset prices are a signal of value—. This model is consistent, in particular, with the fact that prices rise gradually but fall suddenly, and that there are more price runs than predicted by random chance. These studies show that although testing for bubbles is difficult in general (Flood and Hodrick (1990)), there are some indirect ways of looking at the data that reveal inconsistencies with the fundamentalist view

[17] In the sense, probably, of sistematic deviations of subjects' beliefs from 'objective' distributions.

and at the same time could be assimilated to some particular forms of irrationality.

Survival

An important point relates to the survival in the market of "wrong models" of this sort. Friedman (1953) is generally interpreted as raising this point, when saying:

> "People who argue that speculation is destabilizing seldom realize that this is largely equivalent to saying that speculators lose money, since speculation can be destabilizing in general only if speculators on the average sell when the currency is low in price and buy when it is high." (Essays in Positive Economics, page 175).

To complete the argument, we need to add that those who lose money will be forced to leave the market, thereby only 'rational' or 'right' speculators —who will stabilize prices— will be selected.

The first part of the argument, the one established by Friedman, has been proven wrong from a logical perspective in a number of studies, notably Hart and Kreps (1986). The authors construct couterexamples to it, basically replacing the idea that rational speculators "buy cheap and sell dear" with "buy when the chances of appreciation are high, and sell when the chances of depreciation are high", which obviously may or may not be when prices are high and low, respectively. Thus, speculators may make money —and therefore survive— and destabilize prices.

Similarly, De Long et al. (1991) show, in a context more closely connected to our previous idea of irrational traders, that those traders may not only survive, but even dominate the market, eventually. Indeed, the group of overconfident investors —that is, those who underestimate risks— will risk more; as long as the market rewards risk-taking, its wealth may increase over the long run even up to the point of dominating the group of rational investors, and despite of the fact that they are more likely to become ruined and that their overconfidence makes them consume more. Kyle and Wang (1997) raise an additional source of supremacy in the context of a duopoly game: overconfident investors may outperform rational investors because overconfidence may act like a commitment device.

1.4.2 Speculative attacks

The issue of speculative attacks is somewhat different from the problem of the possibility of speculation addressed in section 1.3, for in this case the gains to speculators would come from a government and, as long as it does not maximize "profits", the zero-sum feature of speculation is not present. Thus, the literature on speculative attacks was able to make progress using the rational expectations approach despite the no-trade theorems.

One example of such a model is in Salant (1983). Salant's concern is to explain the failure of governmental attempts to stabilize commodity prices by using buffer stocks. The main point is that an attempt to peg the price will eventually fail, for speculators will correctly anticipate situations where the stock is insufficient to maintain the policy, and rationally buy the remaining stock in a short period of time. Flood and Garber (1994) have used similar arguments to discuss attacks on currencies. Speculation, in this context, refers to anticipating price changes due to governmental policy inconsistency.

1.5 Concluding remarks

The information-based literature on speculation has focused on its compatibility of the rationality assumption. A first avenue lead to the conclusion that speculation is an incomplete-market phenomenon, since differences in belief explanations to trade could be completely exhausted in just one round of trade, if markets were complete. However, there is the belief that the amount of trade explained in this fashion falls short as compared to actual trading volume in asset markets. This approach implies that speculation improves welfare, since it serves as a substitute for missing markets.

What followed was an attempt to explain belief heterogeneity by differences in information. The result, however, pointed in the opposite direction. Rational agents cannot bet against each other based exclusively on private information. As a consequence, prices must reflect all available information, guiding correctly the allocation of resources. Asset pricing models were constructed typically over the assumption of belief homogeneity.

The finance literature understood, then, that analyzing informational issues requires breaking full revelation of prices. This role was taken by the so-called noise traders, presumably rational traders with unpredictable behavior. Ironically, under this approach regular investors were seen as creating noise in an otherwise pure speculative market, switching roles with speculators, who in the traditional literature were the (not always welcomed) visitors.

This literature turned out to be very productive. However, several empirical anomalies started to become apparent, like the excess volatility puzzle and the equity premium puzzle. Although we cannot talk about consensus, the incompatibility of these findings with the popular models of asset pricing has gained wider acceptance. In particular, the predominant interpretation is to see them as limitations of models based on rationality.

Recently, some models based on specific forms of irrationality, notably overconfidence, have shown to explain many of the existing anomalies. Moreover, evolutionary arguments have challenged a conjecture previously rooted in the profession, namely, that irrational behavior could not survive. These ideas naturally revive the interest on welfare effects of speculation. For instance, what Keynes thought about liquidity as a promoter of speculation sixty years ago, did not make sense under rational expectations but could be compatible with the recent twist in asset pricing:

> "The only radical cure for the crisis of confidence which afflict the economic life of the modern world would be to allow the individual no choice

> between consuming his income and ordering the production of the specific capital-asset which, even though it be on precarious evidence, impresses him as the most promising investment available to him. [...] But that would avoid the disastrous, cumulative and far-reaching repercussions of its being open to him, when thus assailed by doubts, to spend his income neither on the one nor on the other." (Op. cit., page 161).

It is too early to assess whether irrationality-based models do a better job than rationality-based models in predicting investor behavior and market outcomes; and even more to assess the desirability of promoting financial systems not based on liquidity creation. However, from an empirical perspective, this is a promising avenue. And from a theoretical perspective, it shows the need for more general models of economic survival, and for welfare concepts capable of including individuals who exhibit particular forms of irrationality as well.

Chapter 2

Informational Efficiency and Competition

2.1 Introduction

Asset prices are usually thought to summarize all payoff-relevant information that is economically available to market participants. In this view, every trader appraises the chances of price appreciation or depreciation and dividend changes, and translates that appraisal into a decision that, collectively, determines the price. However, when modeling such idea a surprising result is readily found, which we refer to as the paradox of informationally efficient markets.

In order to have an incentive to trade based on private information, one must believe that the particular piece of information at hand is not "included" in the price.

But if everybody in possession of news actually were to trade, then the price of the asset would "reflect" all that private information. Moreover, it would reflect more and better information than what any individual trader might possess. Indeed, just by observing the price, that better information could be inferred by everybody, making the formerly private information freely available to all market participants. Thus, the paradox of informationally efficient markets relates to the problem of the incentives to use information.

Yet, this paradox obtains in Walrasian economies, characterized by the fact that the price at which all transactions are carried out is publicly observed, and where unrestricted competition makes the law of one price hold. This picture corresponds to centralized markets, like the ones organized around exchanges.

There are, however, other asset markets that do not operate in this centralized fashion. Let us consider, for instance, the foreign exchange market: most of the transactions take place in private, bilateral agreements, and the marketplace is really a phone network. In cases where transactions take place in this manner, it is not clear how information will be included in prices.

To study this issue, we analyze a random matching model where both, informed and uninformed sellers, make take-it-or-leave-it offers to anonymous buyers, which could also be informed or uninformed. In this setting, the price cannot have any informational content, for every type of trader should make the same offer if she does not want to either reveal herself or be easily imitated by someone who prefers to hide behind her. Whoever deviates by making a different offer, either reveals information —thereby making it worthless— or provides the incentive to be imitated by the type she wants to separate from. The only sequential equilibrium is a pooling one, with a unique price. As a consequence, the price entails no further information than what is commonly known.

Although this result is robust to the informational structure, there is another sense in which it is not: competition. In effect, it holds in a situation characterized by absence of competition, since individuals cannot affect the probability of being in the market even if they are willing to pay a high price for it.

This last thought suggests that it is competition what makes prices carry information. In fact, the feature of the model that drives the result is that the matching is completely random, not taking into account the individual valuation of being in the market. In that sense, the market is seen as a collection of pairwise rather than collective agreements. When competition is introduced by auctioning the right to be in the market, full revelation obtains.

There are several contributions that analyze the connection between market structure and price revelation. Full revelation is proved to obtain in Walrasian settings; for instance, in Milgrom and Stokey (1982) and Tirole (1982) from a general equilibrium perspective, and in Kreps (1977) from a partial equilibrium point of view. Imperfect competition models include Kyle (1985), who analyzes the pricing policy of a monopolistic market maker, and Glosten (1989), who compares the consequences of determining prices monopolically as above, versus by a zero-profit condition, as would be expected in a perfectly competitive market-makers market.

This paper departs from that literature in that the asset market itself is assumed to be imperfectly competitive, as a consequence of having traders negotiating bilaterally rather than all together as would be the case in a centralized marketplace.

There are other articles with this approach, notably Wolinski (1990), who studies the informational content of prices' evolution when the market meets repeatedly, and Hopenhayn and Werner (1996), who show the connection between information and asset liquidity. The former, however, assumes a particular form of bargaining that precludes an equilibrium of the kind we find here, because it allows that some agents get negative surplus. The latter, on the other hand, assumes pre-determined trading conditions.

The rest of the paper is organized as follows: section 2.2 introduces the model, section 2.3 is devoted to the analysis of equilibrium, while section 2.4 introduces competition, and section 2.5 contains the concluding remarks.

2.2 The model

The decentralized market will be represented by a random matching model. There is a continuum of risk-neutral agents, which can be classified into *utility traders* and *speculators*. Utility traders are uninformed players that get utility from trading, that is, they would be willing to trade even if they faced the expectation of a capital loss of up to a certain level x. Half of them receive utility from buying, half of them from selling. Speculators, in turn, are those players that would trade only in the expectation of a capital gain. Thus, their expected utilities can be written as

$$u = (x * \mathbf{1}_{[\text{utility traders}]} + \text{expected value of the bet}) * \mathbf{1}_{[\text{bet}]} \tag{2.1}$$

where $\mathbf{1}_{[\text{utility traders}]}$ and $\mathbf{1}_{[\text{bet}]}$ are indicator functions, that take on the value 1 in the case of utility traders and when the bet is carried out, respectively, and 0 otherwise. Utility traders represent a fraction $z\%$ of the population.

At the beginning of the game, speculators receive a message from nature —possibly the same across speculators— which translates into a higher or lower probability of a higher asset price in the future. With this information at hand, they decide whether to become buyers or sellers. At that point, buyers are randomly matched to sellers.

However, if supply and demand are of a different size, not all agents will have the chance to trade. If an agent is not matched with anybody else, her payoff is zero. When there is a meeting, the seller makes a take-it-or-leave-it offer to the buyer, specifying the price of the transaction. No one is allowed to trade more than one unit. If the offer is accepted, the asset is transferred at the proposed price, while if it is rejected, they both get zero.

There are two payoff-relevant states of nature, θ_1 and θ_2, that have the interpretation of the value of the asset going up or down, respectively. Information is modeled in the usual way, that is, there are basic messages or signals $\eta \in \Lambda$, which are going to be (imperfectly) observed by agents. The probability that a message η is sent by nature is $\mu \equiv \Pr(\eta)$. The composed messages $E \in 2^{\Lambda}$ are called *informational events.* What events are known to a player at different times is described by his particular information partition. The *informational structure* of the game is a description of the information partitions each type of player has, $\{H_\tau\}_{\tau=1}^{\Upsilon}$.

A basic message translates into a probability of a higher future price according to $\pi(\eta) = \Pr(\theta_1|\eta)$. Since the probability of event E is $Pr(E) = \sum_{\eta \in E} \mu(\eta)$, we have

$$\pi_E \equiv \Pr(\theta_1|E) = \frac{\sum_{\eta \in E} \pi(\eta)\mu(\eta)}{\sum_{\eta \in E} \mu(\eta)} \tag{2.2}$$

If a buyer, knowing that the informational event E occurred, engages in a bet $(1-\alpha, -\alpha; \Pr(\theta_1|E), \Pr(\theta_2|E))$, her expected monetary gain is given by

$$\begin{aligned} g_E(\alpha) &= \Pr(\theta_1|E)(1-\alpha) + \Pr(\theta_2|E)(-\alpha) \\ &= \pi_E - \alpha \equiv \alpha_E^* - \alpha \end{aligned} \tag{2.3}$$

Without loss of generality, assume that the basic signals are ordered, so that $\pi_N \leq ... \leq \pi_2 \leq \pi_1$. Then, we can put in a line all fair bets —those that give an

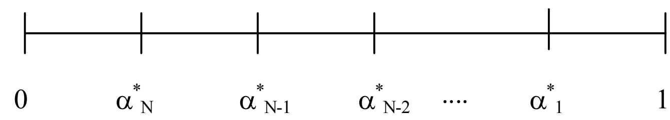

Figure 2-1: Fair bets for each possible message

expected utility of zero— as figure 2-1 shows.

Sellers come in many types $\{\tau\}_1^{\Upsilon}$, each one of them associated to a different information partition $\{H_\tau\}_1^{\Upsilon}$. A strategy for a type-τ seller is a function that associates an offer to each informational event, according to:

$$\alpha_\tau : H_\tau \longrightarrow \alpha(h_\tau(\eta))$$

Thus, $\alpha_\tau(\eta)$ is the offer made by an informed seller when the signal is η and, consequently, she knows $h_\tau(\eta)$. Similarly, we will write $\alpha(\Lambda)$ to denote the offer made by an uninformed seller, that is, by the one who only knows that something happened.

The resulting strategies give rise to a probability distribution over offers (α), where the probability of a particular offer corresponds to the probability of all messages that would lead to that offer:

$$\delta^i : H_\tau \longrightarrow \Delta\{\alpha\}$$

Collectively, sellers generate

$$\delta : \Lambda \longrightarrow \Delta\{\alpha\}$$

specifying for each $\eta \in \Lambda$[1] a probability distribution $\delta(\alpha, \eta)$ over the set of possible offers, that is,

$$\delta(\alpha, \eta) = \sum_{\tau} \gamma_\tau \delta_\tau(\alpha, \eta)$$

Buyers respond to each offer by accepting or rejecting it. A buyer of type τ will find herself typically in an information set composed of an offer α and a signal $h_\tau(\eta)$:

$$\sigma_\tau : H_\tau \times \{\alpha\} \longrightarrow [0, 1]$$

where $\sigma(\alpha, \eta) \in [0, 1]$ is the probability of accepting an offer α in state η. The collective choice of buyers is given by

$$\sigma : \Lambda \times \{\alpha\} \longrightarrow [0, 1]$$

where

$$\sigma = \sum_{\tau} \gamma_\tau \sigma_\tau(\alpha, \eta)$$

The equilibrium will be characterized, then, by a probability distribution over signals, offers and answers.

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[1] Actually, the strategy has as a domain the join of $\{H_\tau\}_1^\Upsilon$ and not Λ, but it is written this way for notational simplicity.

Sellers of type τ that offer $\alpha(\eta)$ have an ex-ante utility of:

$$\sum_{\eta} \mu_{\eta} \left[x + \alpha^*_{\eta} - \alpha(\eta)\right] \sigma(\alpha; \eta)$$

where $h_{\tau}(\eta) = h_{\tau}(\eta') \Rightarrow \alpha(\eta) = \alpha(\eta')$.

Similarly, buyers of type τ have an ex-ante utility of:

$$\sum_{\eta} \mu_{\eta} \left[x + \alpha - \alpha^*_{\eta}\right] \delta(\alpha, \eta)$$

Definition 1 *An equilibrium for this game is a pair* $(\sigma(\alpha; \eta), \delta(\alpha, \eta))$ *where*

$$\begin{aligned}
\sigma(\alpha, \eta) &= \sum_{\tau} \gamma_{\tau} \sigma_{\tau}(\alpha, \eta) \text{ and} \\
\sigma_{\tau}(\alpha, \eta) &\in \arg\max \sum_{\eta} \mu_{\eta} \left[x + \alpha - \alpha^*_{\eta}\right] \delta(\alpha, \eta) \\
\delta(\alpha, \eta) &= \sum_{\tau} \gamma_{\tau} \delta_{\tau}(\alpha, \eta),\ \delta_{\tau}(\alpha, \eta) \\
& \quad \text{is derived from } \alpha_{\tau}(\eta) \text{ and} \\
\alpha_{\tau}(\eta) &\in \arg\max \sum \mu_{\eta} \left[x + \alpha^*_{\eta} - \alpha(\eta)\right] \sigma(\alpha; \eta)
\end{aligned}$$

2.3 Equilibrium

The equilibrium of this game is a pooling one, that is, all types of sellers make the same offer irrespective of the information received.

Information Sets	Expected Utilities
Sellers:	
$\{\eta_1\}$	$(\alpha(\eta_1) - \alpha_1^*)\sigma(\alpha(\eta_1), \eta_1)$
$\{\eta_2\}$	$(\alpha(\eta_2) - \alpha_2^*)\sigma(\alpha(\eta_2), \eta_2)$
$\{\eta_1, \eta_2\}$	$\mu_1[x + \alpha(\Lambda) - \alpha_1^*]\sigma(\alpha(\Lambda), \eta_1) + \mu_2[x + \alpha(\Lambda) - \alpha_2^*]\sigma(\alpha(\Lambda), \eta_2)$
Buyers:	
$\{\eta_1\}, \alpha$	$(\alpha_1^* - \alpha)\delta(\alpha, \eta_1)$
$\{\eta_2\}, \alpha$	$(\alpha_2^* - \alpha)\delta(\alpha, \eta_2)$
$\{\eta_1, \eta_2\}, \alpha$	$\mu_1[x + \alpha_1^* - \alpha]\delta(\alpha, \eta_1) + \mu_2[x + \alpha_2^* - \alpha]\delta(\alpha, \eta_2)$

Table 2.1: Expected utility of an offer

We will verify the validity of this claim by looking at special cases of informational structures in increasing complexity.

Informational Structure 1: $\Lambda = \{\eta_1, \eta_2\}$, $H_{UT} = \{\eta_1, \eta_2\}$, $H_S = \{\{\eta_1\}, \{\eta_2\}\}$

In this first case there is only one type of speculator, who knows precisely what the message was, and therefore cannot learn from anything. The expected utilities are as in table 2.1.

The first thing to note is that when $\alpha(\eta_1) \neq \alpha(\eta_2) \neq \alpha(\Lambda)$, the information will be completely revealed. Thus, from an informational point of view, the knowledge of the offer will be redundant for speculators while for utility traders it will convey information (unless the seller was also uninformed).

Consider, then, the situation of a seller upon receiving message $\{\eta_1\}$. In principle, he could face another speculator that accepts up to $\alpha(\eta_1) \leq \alpha_1^*$ in which case there is no way of profiting. But he could also face a utility trader, who would accept up to $\alpha \leq x + \alpha_1^*$. The best he could do, then, is to offer $\alpha(\eta_1) = x + \alpha_1^*$. Similarly,

$\alpha(\Lambda)$	Expected utility	if
α_2^*	$x - \mu_1[\alpha_1^* - \alpha_2^*]$	
$x + \alpha^*$	$2x - (1-z)\mu_2[2x + \alpha^* - \alpha_2^*]$	$x + \alpha^* \leq \alpha_1^*$
$x + \alpha^*$	$2xz$	$x + \alpha^* \geq \alpha_1^*$
α_1^*	$x + \mu_2[\alpha_1^* - \alpha_2^*] - (1-z)\mu_2[x + \alpha_1^* - \alpha_2^*]$	$x + \alpha^* \leq \alpha_1^*$
α_1^*	$\mu_1 x(1-z)$	$x + \alpha^* \geq \alpha_1^*$

Table 2.2: Expected utility of the maximum offers

upon receiving message $\{\eta_2\}$, his best choice is $\alpha(\eta_2) = x + \alpha_2^*$.

Under the maintained assumption that all three offers are different, the utility traders' offer will be accepted by all types if $\alpha(\Lambda) \in [0, \alpha_2^*]$, by utility traders and speculators when the message is η_1 if $\alpha(\Lambda) \in (\alpha_2^*, x + \mu_1\alpha_1^* + \mu_2\alpha_2^*]$ [2] (if $x + \mu_1\alpha_1^* + \mu_2\alpha_2^* \leq \alpha_1^*$), and only by utility traders if the message is η_2 and $\alpha(\Lambda) \geq \alpha_2^*$ or if $x + \mu_1\alpha_1^* + \mu_2\alpha_2^* \geq \alpha_1^*$.

Let $\alpha^* \equiv \mu_1\alpha_1^* + \mu_2\alpha_2^*$. Taking the maximum of each interval as the offer, they would yield an expected utility according to table 2.2.

When x is low ($x \leq \mu_1[\alpha_1^* - \alpha_2^*]$ and $x \leq \alpha_1^* - \alpha^*$), to sell cheap generates losses. The highest price is the optimal strategy if $x(1-(1-z)\mu_2) \leq \mu_2[\alpha_1^* - \alpha_2^*] - (1-z)\mu_2[\alpha_1^* - \alpha^*]$. However, when x is high ($x \geq \mu_1[\alpha_1^* - \alpha_2^*]$ and $x \geq \alpha_1^* - \alpha^*$), α_2^* might be the best offer.

[2] The value that gives zero utility to buyer utility traders when such an offer could only have been made by another utility trader is $x + \mu_1\alpha_1^* + \mu_2\alpha_2^*$.

This situation is not an equilibrium, though. A seller speculator with "bad news" ($\{\eta_2\}$) could pretend to have good news ($\{\eta_1\}$) by making the offer $\alpha(\eta_1) = \alpha_1^* + x$ and fool a utility trader; after all, he would not sell to another speculator anyway. This misrepresentation incentive is always present, so that we will have $\alpha(\eta_1) = \alpha(\eta_2)$.

Such a situation, however, requires too high prices to keep speculators with good news selling. This in turn imposes a high cost on utility traders, which we assume they are not willing to bear, that is, we will assume throughout that x is not so high as to make utility traders accept trades they know are unfavorable to them, that is,

Assumption $x \leq 1 - \alpha_2^*$.

Alternatively, a speculator with good news may try to become a buyer. This will be an equilibrium if the price charged by utility traders is not so high as to eliminate profits. We contend this is indeed the case:

Proposition 1 *If x is low, then there exists a unique sequential equilibrium in which when the information is $\{\eta_1\}$, speculators take the demand side; when it is $\{\eta_2\}$, they take the supply side; all matched sellers make the offer $\alpha(\eta_2) = \alpha(\Lambda) = x + \alpha_1^* \left(\frac{\mu_1 z}{\mu_1 z + \mu_2}\right) + \alpha_2^* \left(\frac{\mu_2}{\mu_1 z + \mu_2}\right)$, which is accepted by all buyers, and everybody holds consistent beliefs.*

Proof. We first prove that the described strategies conform a sequential equilibrium. Suppose that when the information is $\{\eta_1\}$, speculators take the demand side and that when it is $\{\eta_2\}$, they take the supply side. Then, an uninformed buyer will accept it as long as his expected utility (which considers the adverse selection effect of facing speculators only when the message was $\{\eta_2\}$) is greater than zero, that is, when

$$\begin{aligned} Eu &= \mu_1[x+\alpha_1^* - \alpha(\Lambda)]\delta(\alpha,\eta_1) + \mu_2[x+\alpha_2^* - \alpha(\Lambda)]\delta(\alpha,\eta_2) \geq 0 \\ &\Leftrightarrow \mu_1[x+\alpha_1^* - \alpha(\Lambda)]z + \mu_2[x+\alpha_2^* - \alpha(\Lambda)] \geq 0 \\ &\Leftrightarrow x + \alpha_1^*\left(\frac{\mu_1 z}{\mu_1 z + \mu_2}\right) + \alpha_2^*\left(\frac{\mu_2}{\mu_1 z + \mu_2}\right) - \alpha(\Lambda) \geq 0 \end{aligned}$$

This imposes a limit on the offer a utility trader can make. On the other hand, he will maximize his utility by charging the maximum allowed (which corresponds to the corrected expected monetary gain plus their value from trading), since the acceptance is not sensitive to this price within this limit. Speculators with bad news, on the other hand, are better served by becoming sellers (they would lose if they bought at that price), and cannot charge a different price without being discovered.

To see that the equilibrium is unique, it suffices to realize that no matter what the probability of acceptance or the probability of receiving a particular offer, the expected utility of a buyer speculator will always have the opposite sign than the seller's. Thus, all speculators will choose the same side of the market (unless their expected utility is zero, but that will not happen because of their informational advantage). That leaves us with $x + \alpha_1^*\left(\frac{\mu_1 z}{\mu_1 z+\mu_2}\right) + \alpha_2^*\left(\frac{\mu_2}{\mu_1 z+\mu_2}\right)$ as the unique price. ■

Notice that the equilibrium price does not coincide with $E(\alpha|\{\eta_1, \eta_2\}) = \alpha_1^*\mu_1 + \alpha_2^*\mu_2$. On the one hand, there is the extra component "x", reflecting the strong bargaining power the structure of the game gives to sellers. On the other hand, the weights on the expectation are different, which is explained by the adverse selection effect. As a matter of fact, it is the chance of facing a speculator what corrects the expectation, moving it towards the worst-case scenario as the proportion of speculators increases in the population.

This fact can be illustrated as follows: suppose a speculator believes that the price will drop. He will never take the demand side unless the buyer is willing to pay such a high price that the information is actually valueless. In such a case, only the position of seller is valuable, regardless of the information at hand. This situation was ruled out by assumption (x low).

One might wonder if more complicated informational structures would change the nature of equilibrium. We explore that problem in the next two versions of the model.

Informational Structure 2: $\Lambda = \{\eta_1, \eta_2, ..., \eta_N\}$, $N > 2, H_{UT} = \{\Lambda\}, H_S = \{\{\eta_n\}\}_{n=1}^N$.

This informational structure is slightly more complicated, but the previous result carries over without difficulty. What is different now is that there are many price expectations speculators might hold, and consequently various degrees of intensity of the willingness to trade they could have. However, intensity of desire does not translate into intensity of action, because of risk neutrality. Therefore, from the point of view of a utility trader, it does not make any difference, except for the fact that he would now modify his reservation price by a slightly more complicated adverse selection cost.

Proposition 2 *If x is low, then there exists a unique sequential equilibrium in which when the information $\{\eta_n\}$ points to an $\alpha_n^* \leq \alpha(\Lambda)$, speculators take the demand side; when it points to an $\alpha_n^* \geq \alpha(\Lambda)$, they take the supply side; all matched sellers make the offer*

$$\alpha(\eta_i) = \alpha(\Lambda) = x + \sum_{\eta_i \in G} \left(\alpha_n^* \frac{\mu_i z}{\sum\limits_{i \in G} \mu_i z + \sum\limits_{i \in B} \mu_i} \right) + \sum_{\eta_i \in B} \left(\alpha_i^* \frac{\mu_i}{\sum\limits_{i \in G} \mu_i z + \sum\limits_{i \in B} \mu_i} \right)$$

(where $\widetilde{\eta}$ is defined by $\alpha^*(\widetilde{\eta}) = \alpha(\Lambda)$, $G \equiv \{\eta_1, ..., \widetilde{\eta}\}$ and $B \equiv \{\widetilde{\eta}, ..., \eta_N\} = \Lambda \backslash G$), which is accepted by all matched buyers, and everybody holds consistent beliefs.

Proof. Similar to that of proposition 1. ■

Informational Structure 3: $\#\Lambda = N > 2$, $H_{UT} = \{\Lambda\}$, $H_S \in \{H_\tau\}_{\tau=1}^{\Upsilon}$.

Speculators now receive different (composed) messages, while utility traders are as before. Each type of speculator is represented by a different partition of $\Lambda = \{\eta_1, \eta_2, ..., \eta_N\}$. This has the interpretation that when the message sent is η_i, a type-m speculator will know that the event $h_m(\eta_i)$ occurred. Each type is actually informed of a different fact, and this is common knowledge.

In this case there is a qualitative difference: it is possible that speculators trade with each other, since the existence of utility traders prevents learning the identity of the opponent from their actions (the Milgrom-Stokey type of reasoning). Needless to say that how frequently this would happen depends on the actual form of the partitions and the distribution of messages.

Still, our main conclusion above remains a valid one: Speculators will choose their side on the market based purely on their private information, and those who become sellers will make the same offer an uninformed investor would.

Proposition 3 *If x is low, then there exists a sequential equilibrium in which when the information $\{\eta_i\}$ points to an $\alpha^*_{h(\eta_i)} \leq \alpha(\Lambda)$, speculators take the demand side; when it points to an $\alpha^*_{h(\eta_i)} \geq \alpha(\Lambda)$, they take the supply side; all matched sellers make the offer $\alpha(h(\eta_i)) = \alpha(\Lambda)$, which is accepted by all matched buyers, and everybody holds consistent beliefs.*

2.4 Introducing competition

This section modifies the model to introduce competition in the form of a market for being matched. Suppose that prior to being matched, people have to buy a position either as a buyer or as a seller. A Walrasian market for positions would require the probability of a match to be 1 for buyers and sellers, that is, market clearing. In a more general way, we could say that the mechanism auctions off a given probability of being matched.

Let ρ_s and ρ_b be the probabilities that a seller and a buyer get matched, respectively, and let $P(\rho_s)$ and $P(\rho_b)$ be the prices they pay for such probabilities. In general, these prices may depend on the signals, $P(\cdot, \eta)$.

Consider the case where $\rho_s = \rho_b = 1$ (Walrasian). When news are bad, speculators rush to sell, bidding up selling positions, and creating a surplus of buying positions. When news are good, buying positions should be expensive and selling positions cheap. But this implies that the price of a particular position fully reveals the message.

This is indeed an equilibrium. When news are bad, $P(\rho_b, \eta_2) = 0$ and $\rho_b = 1$. Buyers infer that $\eta = \eta_2$, and adjust their demands to $\alpha_2^* + x$. Sellers know that if they are matched they will get $\alpha_2^* + x$. Speculators, then, are willing to pay at most $\alpha_2^* + x$ for a seller position, while utility traders $\alpha_2^* + 2x$. Thus, seller positions sell for $P(\rho_s, \eta_2) \in \alpha_2^* + x, \alpha_2^* + 2x]$, and $\rho_s = 1$ thereby completely excluding speculators.

Similarly, when there are good news, $P(\rho_s, \eta_1) = x = P(\rho_b, \eta_1)$, $\alpha = \alpha_1^*$, and $\rho_b = 1 = \rho_s$. Actually, as $\alpha = \alpha_1^* + x - P(\rho_b)$ discourages speculators from buying for any $P(\rho_b) \geq \frac{x}{2}$[3], it suffices that they do not have incentives to become sellers[4]. In this equilibrium, utility traders trade with utility traders and prices are informative.

It is easy to see that the same reasoning applies to the more complex informational

[3]Since buyers' surplus is $\alpha_1^* - \alpha_1^* + x - 2P(\rho_b) \leq 0$ if $x - 2P(\rho_b) \leq 0$.

[4]If they do, they get $\alpha_1^* + x - P(\rho_b) - \alpha_1^* - P(\rho_s) \leq 0$, which is, $x \leq P(\rho_b) + P(\rho_s)$.

structures studied before. The essential feature is that all investors participate together in forming the price. As long as the publicly known price is affected by market conditions, it reveals them.

2.5 Concluding remarks

In the previous chapter, we saw that full information revelation obtained under any voluntary trading arrangement, provided that we had common priors, a Pareto-optimal initial allocation and common knowledge of actions. In this chapter, we have departed from the above on the optimality of the original resource allocation.

Our main conclusion is that the extent to which private information is revealed in the trading process depends crucially on how competitive is the environment in which traders interact. In Walrasian economies, in particular, prices are revealing because of the extreme competition level embedded on them.

Chapter 3

On the limits to speculation in centralized versus decentralized market regimes

3.1 Introduction

If speculation, or information-based trading, is to be profitable, it must be at the expense of regular traders or investors, which we term *utility traders*. Utility traders use asset markets for non-specific purposes, usually categorized as consumption smoothing, insurance, investment, etc. Even though markets are beneficial for them, they will choose not to participate in the event that the adverse selection cost imposed by the action of speculators exceeds the benefits of using the market.

Nevertheless, as it is widely known, the market requires utility traders to operate, since private information alone is not sufficient to create trade, that is, a market composed solely of speculators will be characterized by zero volume (no-trade theorem[1]). Therefore, we conclude that there is a limit to the amount of speculative transactions that a given market can afford, relative to non-speculative transactions that take place. If this ratio crosses that border, transactions will be zero, exactly as if there were no utility traders at all, and the no-trade theorem would apply. That ratio is determined by the maximum rents that can be extracted from utility traders before they abandon the market.

We explore this intuition: the existence of a market depends on the composition of its participants, according to their motivations for trading. Moreover, we ask how these limits vary across different market regimes. In particular, we compare a centralized (intermediated) market regime to a decentralized (non-intermediated) one, finding that the former is more prone to speculation. Our model tells us that the key issue determining this is the ability that an eventual intermediary has for transferring utility from the incumbent speculators to new ones —an ability generated by the act of charging transaction fees as a method of collecting profits. In fact, they provide a mechanism to diminish the individual return to information, decreasing the informational rent, so that for a fixed value of trading or surplus, more speculative

[1] For a version of this theorem, see Milgrom and Stokey (1982).

transactions can be supported.

The analysis also opens two branches: on the one hand, it suggests a reason for the existence of intermediaries in financial markets, based on the adverse selection cost that the uninformed bear when they trade with the informed. This is unrelated to the incentive problems advanced by Leland and Pyle (1977). On the other hand, it allows the study of the conditions under which there will be a spontaneous move towards intermediation, or desintermediation. Both branches are briefly discussed at the end.

To address these issues, we use a random matching model in which players are paired to voluntarily bet on the occurrence of two states. Trade is modeled by the simultaneous acceptance of a bet. By modeling a betting game rather than a game in which players actually trade an asset, we hope to simplify the analysis while capturing what is essential to it. The key observation is that ultimately, any decision of buying or selling an asset involves a bet: whoever buys is betting that the price will not drop the following day, whoever sells is betting on the opposite. Regardless of the particular reasons any person could have to buy or sell an asset, the decision of doing it today rather than tomorrow reveals certain level of trust on the favorability of today's conditions over tomorrow's: that is where the bet lies. What we are missing in the simplification is the fact that people may actually choose which side of the market they want to be in, but this amounts to say that the bets are endogenous. We will discuss some methodological issues at the end.

To summarize, then, our main results are:

1. Given a certain value from trading, there is a maximum amount of speculative activity that a decentralized market can sustain. If the proportion of speculative over non-speculative bets passes that limit, the market shuts down (no-trade region).

2. That limiting amount is zero in an economy with a unique intermediary, that is, the intermediary is always able to keep the market open.

3. Moreover, the intermediary provides higher liquidity and volume is greater than in the decentralized market, increasing welfare.

The present research is connected with two areas. On the one hand, we have the adverse selection problem that the uninformed face when trading with the speculators, or informed –a lemons problem–. Milgrom and Stokey (1982) provided an example of the "no trade theorem," example on which the present model is based. However, they took the view that the theorem implied the incompatibility of the rational expectations model with reality. Glosten (1989), in a different setting (actually, the standard in the finance literature) analyzes the differences between competitive vs. monopolistic market makers. He concludes that when the asymmetries are more severe, the monopolist is better because it increases liquidity, since it is not forced to make zero profits on each transaction. However, he does not consider the equilibrium without market makers, nor does he analyze the limiting amount of speculation.

Bhattacharya and Spiegel (1991) study the equilibria with an informed monopolist and a continuum of uninformed risk averse traders, in a setting similar to Glosten's.

A second literature, from finance, refers to noisy rational expectations equilibria. A seminal paper is Grossman and Stiglitz's (1980). We do not address the issue of information revelation; rather, we explicitly incorporate the role of the noise traders as utility traders. Utility traders are simply individuals who place a positive value on exchange. As opposed to noise traders, however, their behavior is endogenous. In that exogenous behavior paradigm, it is the noise that prevents the no-trade result, while in our model it is the surplus they generate what prevents it. The problem of learning here was assumed away, for there is no aggregate statistic about the state of the economy from which the players could infer something. Instead, what we need utility traders for is to generate a rent.

The rest of the paper is organized as follows: Section 3.2 introduces the model. Section 3.3 is devoted to the analysis of the decentralized market, while section 3.4 studies the market with an intermediary. Section 3.5 concludes and discusses possible extensions.

3.2 The model: a betting game

There is a continuum of risk neutral players with common priors. Half of them will be assigned the role of a "buyer", the other half the role of a "seller". There is nothing to buy or sell; the name of "buyer" or "seller" is purely metaphorical. At date 1, every buyer is randomly matched with a seller, and vice versa. Then, the speculators will get to see a signal $\omega \in \{\omega_1, \omega_2\}$ while the utility traders see nothing. At that point, everyone is offered a bet: buyers are offered the possibility of winning \$1 if state θ_1 happens while losing \$1 if θ_2 happens; sellers are offered the complementary bet, that is, the possibility of losing \$1 if state θ_1 happens while winning \$1 if θ_2 happens. In each match, the bet is carried out only when they both accept; if any player, the one in the role of the buyer or the one in the role of a seller, rejects the bet, they both get \$0. After confirming the acceptance, date 2 starts and everybody gets to see the state $\theta \in \{\theta_1, \theta_2\}$ and the payments are carried out. The bet is ex-ante a fair game, that is, the prior probability of θ_1 is 0.5. We will further assume that each signal, ω_1 and ω_2, is equally likely.

The names of speculators and utility traders are assigned depending on the particular form of the utility function of each player. In general,

$$u = (x * \mathbf{1}_{[\text{utility traders}]} + \text{expected value of the bet}) * \mathbf{1}_{[\text{bet}]} \tag{3.1}$$

where $\mathbf{1}_{[\text{utility traders}]}$ and $\mathbf{1}_{[\text{bet}]}$ are indicator functions, that take on the value 1 in the case of utility traders and when the bet is carried out, respectively, and 0 otherwise. This is to say that utility traders enjoy gambling, getting a utility level of $x > 0$ just for betting. However, they are uninformed. On the other hand, speculators are informed but gambling is a neutral for them. Thus, in this model a pure speculator is someone who would not participate if he did not expect a direct monetary gain by betting, while a utility trader is someone who would participate even if she expected up to a certain monetary loss.

Notice that we could have defined four types, instead of two. We omitted the informed that enjoy gambling and the uninformed that regard gambling as a neutral. This exclusion was deliberately made in the sake of simplicity. However, it comes at no cost: these types play no role. Their behavior would be the same as the two types that remained. In addition, it allows us to identify motivations with people, which cannot be done in reality as easily as here.

Although this separation of traders according to their motivations is not something that we could hope to do as easily in practice, there is an argument to identify speculators with better information: if information were costly, speculators would have the highest demands for it, since they are the ones that would get the highest surplus from it. This is so because they are prepared to use information more fully than utility traders, in the sense that the arrival of even weak evidence will change the behavior of a speculator but not the behavior of a utility trader.

Let us say that $\Pr(\theta_1|\omega_1) > 0.5 = \Pr(\theta_1) > \Pr(\theta_1|\omega_2)$, so that a buyer would find it favorable to accept after receiving the signal ω_1. Let "z" be the percentage of utility traders in the total population, and "g" the expected gain for a buyer conditional on receiving a signal ω_1 (our symmetry assumption implies that g is also the expected

	Buyer	**Seller**
Speculator	$(1-z)/2$	$(1-z)/2$
Utility trader	$(z/2)$	$(z/2)$
TOTAL	50%	50%

Table 3.1: The distribution of types.

gain of a seller conditional on receiving a signal ω_2, since $\Pr(\theta_2|\omega_2) = \Pr(\theta_1|\omega_1)$). Then,

$$\begin{aligned} g &= (1)\Pr(\theta_1|\omega_1) + (-1)\Pr(\theta_2|\omega_1) = \frac{\Pr(\theta_1 \wedge \omega_1) - \Pr(\theta_2 \wedge \omega_1)}{\Pr(\omega_1)} \qquad (3.2) \\ &= (1)\Pr(\theta_2|\omega_2) + (-1)\Pr(\theta_1|\omega_2) \end{aligned}$$

The distribution of types is common knowledge, and is as in table 3.1.

Throughout we will assume that $x < g$; otherwise, the utility from gambling would be so high relative to the expected monetary gain/loss, that a utility player would not care about timing his decision. It follows that a speculator in possession of good news will always accept, while in possession of bad news never will: there is nothing else that such a person could learn either by direct observation or by inferring from other people's behavior, that would make him change his mind[2]. He knows whether

[2]This is a consequence of assuming that there is one signal common to all, rather than one for each individual. The latter would be required to analyze the information aggregation problem, which we do not aim to do here.

Upon receiving the signal...	ω_1	ω_2
Buyer		
- Speculator	$g\,\Pr(\text{opponent accepts})$	$-g\,\Pr(\text{opponent accepts})$
- Utility trader	$\left\{\frac{1}{2}(x-g)\,\Pr(\text{opponent accepts if } \boldsymbol{\omega}_1)\right.$ $\left.+\frac{1}{2}(x+g)\,\Pr(\text{opponent accepts if } \boldsymbol{\omega}_2)\right\}$	
Seller		
- Speculator	$-g\,\Pr(\text{opponent accepts})$	$g\,\Pr(\text{opponent accepts})$
- Utility trader	$\left\{\frac{1}{2}(x-g)\,\Pr(\text{opponent accepts if } \boldsymbol{\omega}_1)\right.$ $\left.+\frac{1}{2}(x+g)\,\Pr(\text{opponent accepts if } \boldsymbol{\omega}_2)\right\}$	

Table 3.2: Expected utilities.

the game is fair or unfair to him.

In this way, the problem is in the hands of utility traders: if they do not participate, we get no trade and no market can exist. They will, on the other hand, accept as long as the monetary loss due to the participation of speculators does not outweigh the utility from gambling, x. Then, we have:

We can verify in the table that for a speculator, the expected utility is proportional to g or $-g$, so that the decision is unambiguous, as we claimed earlier. However, this is not true for a utility trader; we analyze her decision in the next section.

Before moving into that, we would like to discuss briefly the probability that the opponent accepts. This probability may depend on the matching rule. So far, we have assumed that the mechanism creates matches before the players get to observe the signal, but it is perfectly possible to conceive, for instance, one in which the mechanism asks about intentions before making matches; in this case, we could have situations in which the largest side of the market gets rationed while the other side is completely served. The same happens here, even though the mechanism does not

	Buyer	**Seller**
Speculators		
ω_1	Accept	Reject
ω_2	Reject	Accept
Utility Traders	?	?

Table 3.3: Who accepts the bet.

try to maximize trade. The reason is that in one of the sides everybody wishes to accept, so that it is always partially rationed. Nevertheless, the conclusions of this analysis will also extend to other matching rules, as long as those rules give rise to probabilities that are proportional to the one we consider, though the utility level of each player will be different.

3.3 Decentralized equilibrium

We now turn to the analysis of who accepts the bet. So far, we know that behavior will be as table 3.3 shows.

We also know that for the market to exist at all, we need utility traders to accept. They will as long as they get positive utility by doing it, that is, as long as

$$\frac{1}{2}(x-g)+\frac{1}{2}(x+g)\geq 0 \Leftrightarrow x\geq g\frac{1-z}{1+z} \Leftrightarrow z\geq \frac{g-x}{g+x} \tag{3.3}$$

There are two possibilities: a utility trader can be matched to another uninformed utility trader, in which case she faces a fair game that is worth accepting, because she gets x. But she could also be matched to a speculator, a case in which she is definitely facing an unfair game, that she clearly would be better off avoiding. As she cannot distinguish a utility trader from a speculator, she would participate if she thinks that it is likely enough that she would find herself in the first situation and not in the second one.

Condition (3.3) embodies the above reasoning, giving a precise meaning to what is "likely enough". The probability of facing an unfair game is determined by the proportion of speculators in the population. Depending on how valuable information is, it will be required a different level of utility x (gains from trade) in order to support trade for a given composition of the population. The more valuable information is, that is, the bigger g is, the higher the adverse selection problem to the uninformed, and as a consequence, the higher the value of trading the asset must be so that she still wants to trade. Alternatively, for a fixed value of x, to maintain trade while increasing g will require a reduction in the proportion of informed.

In other words, the cost of the adverse selection problem to the utility traders is determined together by g, the individual information rent, and $(1-z)$, the probability of being matched with a speculator, that add up to $\frac{1}{2}g(1-z)$. This cost must be smaller than the utility she gets by gambling, x, with probability $\frac{1}{2}(1+z)$.

It is interesting to note that there is a trade-off between the maximum proportion of speculators in the economy and the predictive power of their information. For instance, if this information service is not very accurate, g is small, and the maximum

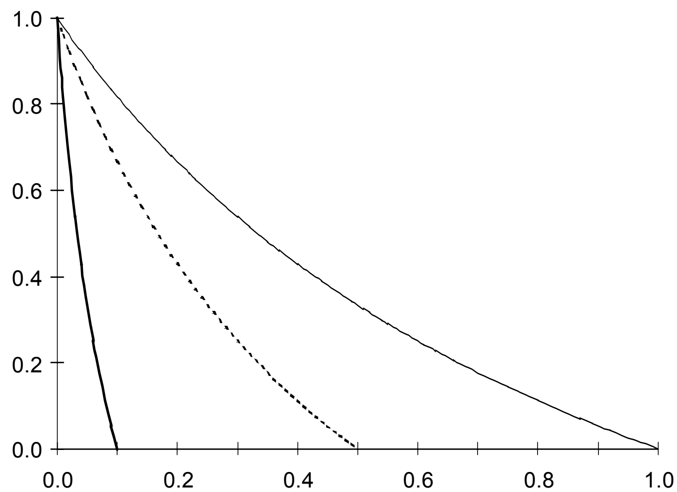

Figure 3-1: Minimum proportion of utility traders as a function of x

number of speculators can be very large with respect to the number of utility traders, that is, a very small proportion of the transactions needs to be non-speculative. This appears to be the case in the foreign exchange market, characterized by a huge volume of trade, many times larger than needs as means of exchange would justify, and traders making many tiny profits on each transaction.

Another way to look at condition (3.3) is this: x and z determine the size of the pie, which in the limiting population composition is completely exhausted by speculators; g is the size of individual portions, i.e., the per-speculator rent. How many of them we can get is a matter of dividing xz by g. The existence of an intermediary will change both, the size of the pie and the size of individual portions.

We can visualize this on figure 3-1. Each line traps below it a “no-trade region”,

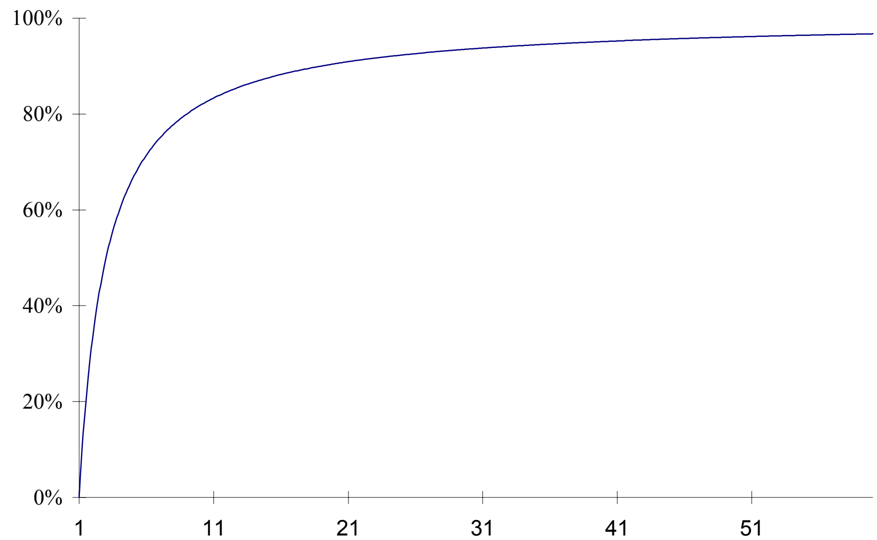

Figure 3-2: Minimum proportion of utility traders as a function of $\frac{g}{x}$.

whose size depends on the value of private information, g. Put another way, the minimum proportion of non-speculative transactions is determined by the potential expected loss relative to the value of owning the asset for one period.

Observe in figure 3-2 the concavity of the function: z becomes nearly insensitive to $\frac{g}{x}$ for high values of this variable.

3.4 One intermediary

Notice that in the decentralized equilibrium utility traders lose to speculators; the only reason why they still trade is that the probability of being matched to play a fair game and therefore gain the utility from gambling overcomes the risk of losing to the better-informed players. Imagine now that one player announces that she will accept all bets, from anyone, no matter what. That single player is telling the uninformed that she will solve their adverse selection problem, so naturally they will prefer to trade with her, rather than in the anonymous decentralized market, even if they are required to pay a small transaction fee. However, if all the uninformed prefer to trade with her, then the decentralized economy is left only with speculators, making the market disappear by the no-trade theorem. Thus, she will centralize all trading, since utility traders prefer to trade with her, while speculators are forced to trade with her when they lose the decentralized market.

In this section, however, we will not address the issue of whether a desintermediated market will move towards intermediation. Rather, we will assume the existence of an intermediary, and we will ask about the maximum speculative activity that such a market can afford.

The first choice variable of this intermediary, that we assume is informed, is to accept or reject a bet from any single player that communicates its intention of betting with her. As bettors are anonymous, except for their roles, this variable takes the form of a probability of accepting to each of them, maybe conditioning on whether she faces a buyer or a seller, and on the message received. The second choice variable is the transaction fee.

We advanced earlier that the transaction fee is the only way the intermediary has to collect profits. The reason is that if she tries to profit from her private information, by giving higher probability of acceptance in the cases in which the public is at a disadvantage, she is replicating the adverse selection problem the utility traders are trying to avoid. To attract them, she must offer better conditions than the decentralized market. However, in this way she collects money only from utility traders, while by charging a transaction fee she will also get money from the speculators, thereby increasing total revenue.

Let y^r_ω be the proportion of bets accepted from role r players ($r = b$ if buyer, $r = s$ if seller) after receiving a signal ω, and let c denote the transaction fee. The problem for this monopolistic intermediary is to maximize profits:

$$\frac{c}{2}\left[\left(\frac{y^b_{\omega_1}}{2}+\frac{zy^s_{\omega_1}}{2}\right)+\left(\frac{y^s_{\omega_2}}{2}+\frac{zy^b_{\omega_2}}{2}\right)\right]+\frac{g}{2}\left[\left(\frac{zy^s_{\omega_1}}{2}-\frac{y^b_{\omega_1}}{2}\right)+\left(\frac{zy^b_{\omega_2}}{2}-\frac{y^s_{\omega_2}}{2}\right)\right] \quad (3.4)$$

subject to the participation of utility traders and speculators[3], that is,

$$\text{s/t } c \leq x+g\left(\frac{y^b_{\omega_1}-y^b_{\omega_2}}{y^b_{\omega_1}+y^b_{\omega_2}}\right),\ c \leq x+g\left(\frac{y^s_{\omega_2}-y^s_{\omega_1}}{y^s_{\omega_1}+y^s_{\omega_2}}\right) \text{ and } c \leq g.$$

Expected profit is, then, composed of the transaction fee that is collected from all buyers and just utility traders among sellers if the information is ω_1, or from all sellers

[3]It is possible for the monopolist to charge $c > g$ by giving back to utility traders the difference $(c - x)$ in the form of accepting more unfavorable bets to herself, thereby excluding speculators completely. However, this strategy is dominated, so it will never be used.

and just utility traders among buyers, if ω_2; plus, the expected payoff formed by the gap between buyers and sellers on each ω, everything weighted by the probability of accepting from a buyer or seller on each state.

We can further simplify the problem by exploiting the symmetry between speculators on each side, as well as utility traders on each side. Let $y_u = y^b_{\omega_1} = y^s_{\omega_2}$ and $y_f = y^s_{\omega_1} = y^b_{\omega_2}$, where subscripts "$u$" and "$f$" stand for unfavorable and favorable trades for the intermediary. Then, the optimization problem of the intermediary can be rewritten as:

$$\max_{\{y_u, y_f, c\}} \frac{1}{2}\left\{y_u(c-g) + y_f z(c+g)\right\} \tag{3.5}$$

$$\text{subject to } c = x + g\left(\frac{y_u - y_f}{y_u + y_f}\right) \text{ and } c \leq g$$

It can readily be seen that the problem of the monopolist is to balance two forces: on the one hand, she would prefer to avoid the adverse selection cost $(c-g)$ by avoiding all unfavorable bets, while accepting all favorable ones; however, moving in such direction minimizes the transaction fee that can be charged and endangers the participation of utility traders.

Observe that to set $y_u = y_f = 1$, that is, to accept all bets, yields positive profits as long as $x \geq g\frac{1-z}{1+z}$, which is precisely the condition for the decentralized market to exist. This is to say that, if the condition for the existence of a decentralized market is met, the condition for the existence of an intermediated market[4] is also met.

Moreover, even when the above condition is not satisfied, it is possible for the intermediated market to exist. In effect, we can verify that when $x = g\frac{1-z}{1+z}$, $\left.\frac{\partial E\pi}{\partial y_f}\right|_{y_u=y_f=1} < 0$ and $\left.\frac{\partial E\pi}{\partial y_u}\right|_{y_u=y_f=1} > 0$, meaning that there is a better strategy than accepting all bets in such case. Therefore, the optimal strategy is able to yield positive profits even in cases in which $x < g\frac{1-z}{1+z}$.

It turns out that the optimal policy takes the form:

$$(y_u, y_f, c) = \begin{cases} \left(1, \frac{x}{2g-x}, g\right) & \text{if } x < g\frac{1-z}{2z} \\ (1, 1, x) & \text{if } x \geq g\frac{1-z}{2z} \end{cases} \tag{3.6}$$

Two elements are noteworthy. First, what is the theme of this paper, the monopolist is able to make profits and keep the market open in any circumstances in which there is some value from trading ($x > 0$). The reason the intermediary is able to keep the market open in situations in which the decentralized market would shut down is that by charging a transaction fee to speculators as well as utility traders, she is able to reduce the individual informational rents, thus allowing a larger number (proportion) of speculators in the population.

Secondly, it is not optimal for the monopolist to use her information "against"

[4]The reader may have noticed that in the paper we only refer to a monopolistic intermediated market, not to any possible intermediated market. Yet, we talk about the existence of intermediated markets in general. The reason for this is that if there are not enough rents for a monopolist to survive, there can be no place for more than one firm.

her customers, in the sense that she will never be more inclined to accept favorable than unfavorable bets to herself. Instead of offering some "adverse selection" to her clients, she will offer some "favorable selection," if any. This is so because that way she increases the transaction fee utility traders are willing to pay, thereby allowing a greater surplus extraction from speculators.

Nevertheless, keeping the market open is not the only difference between these two regimes. There is also a difference on the total number of transactions. In fact, in the decentralized economy only $\frac{(1+z)}{2}\%$ of the possible bets actually take place due to "incorrect" matches, while by not having any matching problem the intermediated market fulfills 100% of the possible matches. This implies that the total surplus generated in the former regime is proportional to $\frac{(1+z)}{2}x$, while it is proportional to x in the centralized market. Therefore, from this perspective the intermediated market is more efficient than the decentralized one.

The increased liquidity also means that the expected utility (before deducing transaction fees) of speculators is higher, an effect that goes in the opposite direction from the transaction fee. It turns out that when the intermediary sets $y_u = y_f = 1$, in our example they cancel out exactly, explaining why the conditions for the existence of a decentralized market and a centralized one in which the intermediary is committed to accept all bets are the same.

The existence of a better strategy than always accepting bets in this limiting case, as shown in our example, explains that the market may still exist under intermediation.

3.5 Concluding remarks

We have compared centralized versus decentralized asset markets in a metaphorical way, by analyzing betting games. Our main conclusion, that a centralized market could exist even when a decentralized one would not, rests on both the fact that by charging a transaction fee the monopolist is able to extract utility from the informed to sustain a larger proportion of them in the population which otherwise would be impossible, and the fact that the monopolist is better suited to deal with asymmetries in the population and the information structure. These ideas go beyond the limited scope of our simple model.

In particular, we have made the following simplifying assumptions.

1. The signals are equally likely (symmetry). If they are not, then the condition for the existence of a decentralized market would be given by the more restrictive of the two participation constraints (the one for the utility traders on the demand side, and the one for the utility traders on the supply side), while the intermediary has the ability of "squeezing" utility traders on both sides simultaneously. The same is true about the composition of speculators to utility traders in both sides of the market, that is, if speculators are more concentrated among buyers, or among sellers. In other words, the monopolist can

exploit asymmetries, either in the population or in the informational structure.

2. Each player cannot bet more than $1. This does not seem to be important, insofar as players are anonymous: we can allow for "larger" players with no substantial change, as long as their bets are bounded.

3. The value of trading is the same across utility traders, and informed traders do not get utility from gambling. Relaxing this assumption would only give continuity to the frontier, leaving the rationale of its existence unchanged.

4. Players do not choose which side of the market they are in. To some extent, this is true, for unless short sales are allowed, not owning the asset clearly defines the side of the bet one can take. However, the same does not hold for someone who owns the asset: without liquidity constraints, it is always possible to take the other side too. In any event, this assumption is restrictive just for utility traders, since we can imagine that it is the same group of speculators that chooses side before being paired rather than two groups being active exchangeably as presented.

5. The populations have the same size. This also seems to be restrictive. We can think of this as meaning that at the current price of the asset –which we do not model– demand equals supply.

A question suggested by the present exercise is: Do we necessarily go from a decentralized to a centralized market? In our example, the answer is on the affirmative, since the condition for the existence of a decentralized market is sufficient to guarantee that an intermediary accepting all bets will get positive profits while offering a transaction fee smaller than the adverse selection cost that utility traders face in the decentralized market. However, we do not know whether the same answer holds in more general cases.

The following questions are, naturally, what would change in the presence of competition, and whether there will necessarily be competitive forces in a centralized market. Those questions are left for future research.

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