Problems on Static Games

1. Dominance and Equilibrium

For each of the following games find 1) all weak and strict dominant strategy equilibria 2) apply iterated **strict** dominance 3) find all pure and mixed Nash equilibria 4) indicate which Nash equilibria are trembling hand perfect and why

a)

2,1	0,0
0,0	1,2

b)

6,6	0,7
7,0	1,1

c)

3,3	2,2	1,1
2,2	1,1	0,8
1,1	8,0	0,0

d)

1,3	1,3
0,0	2,0

2. Dominance and Nash Equilibrium

Prove that a profile is a Nash equilibrium of a game if and only if it is the Nash equilibrium of the game in which strategies have been removed by iterated strict dominance. Prove that a Nash equilibrium of a game in which strategies have been removed by iterated weak dominance is a Nash equilibrium of the original game. Give an example of a Nash equilibrium of a game that is not a Nash equilibrium of the game where strategies have been removed by iterated weak dominance.

3. Correlated Equilibrium

Consider the game

0,0	2,1
1,2	0,0

Show that the correlated strategy profile

1/3	1/3
1/3	0

is in fact a correlated equilibrium

4. Anti-Coordination

Two players must choose whether to specialize – they must choose between being a hunter and a gatherer. After they choose, they meet to play a game. If both are hunters, or both are gatherers, they get no benefit from specialization, and receive a utility of zero. If one is a hunter and one a gatherer, the hunter receives 2 and the gatherer 1 unit of utility. 1) Write the normal form of the game. 2) Find the *symmetric* Nash equilibrium in which both players employ the same strategy. 3) Find a *symmetric* correlated equilibrium (probabilities remain the same when we interchange rows for columns) which Pareto dominates the symmetric Nash equilibrium. The correlated equilibrium may use public randomization if you wish, but you must show it is a correlated equilibrium by showing that neither player wishes to deviate from the recommendation of the randomization device.

5. Trembling Hand Perfection

A strategy profile σ is *trembling hand perfect* if there exists a sequence of strategy profiles $\sigma^n \to \sigma$ with $\sigma_i^n(s_i) > 0$ for all i and $s_i \in S_i$ such that $\sigma_i(s_i) > 0$ implies that s_i is a best-response to σ_{-i}^n . Prove that every trembling hand perfect profile is a Nash equilibrium. Give an example of a Nash equilibrium in a 2x2 game which is not trembling hand perfect and explain why.

6. Becker

There are two groups with k each making a non-negative bid b_k. The utility of group k is

$$u_k = (b_k - b_{-k}) - \beta(b_k - b_{-k})^2/2 - c_k b_k^2/2.$$

Here the interpretation is that the difference in the bids is an inefficient transfer payment between the two groups: the group that bids more gets a higher transfer, this has a social cost determined by β but also higher bids are more costly.

a. show that a Nash equilibrium exists and is unique

b. when is the equilibrium interior?

c. in the interior case compute the Nash equilibrium

c. how do the bids and the transfer $b_k - b_{-k}$ depend on β, c_k?

5. Becker says: higher costs lead to lower bids – is that correct?

6. Becker says: less efficiency leads to lower transfers – is that correct?

7. First Price Auction

Stephen J. Seagull and Clod VandeCamp are the only bidders in an auction on a chinese jacket. The seller does not value the item, but Seagull would pay up to $20,000 for it, while VandeCamp will pay no more than $1,000. Both submit sealed bids, which can be for any of the following amounts: $0.00, $500, $1,000, $10,000, $20,000 or $25,000. In case of a tie, a coin is flipped to see who will get the jacket. Write payoff matrix for this game. What strategies are weakly or strongly dominated? Eliminate weakly dominated strategies, then apply iterated strong dominance. Apply iterated weak dominance.

8. Weak Dominance, Nash Equilibrium, THP (Tuna Dokmeci)

	L	M	R
T	4,?	?,2	3,1
M	3,5	2,?	2,3
B	?,3	3,4	?,2

a) Complete the payoffs such that there is exactly one weakly dominant strategy equilibrium. b) Complete the payoffs such that the game is dominance solvable. Try to make sure a different strategy profile is equilibrium than in part a). Apply iterated elimination of dominated strategies, and explain your steps. c) Take the game you constructed in part a. Find all pure strategy Nash equilibria. Are there any mixed strategy Nash equilibria? Explain. Check if the Nash equilibria you find are trembling hand perfect (THP). Is the equilibrium you found in the first part THP? How about others if there are any? Comment.