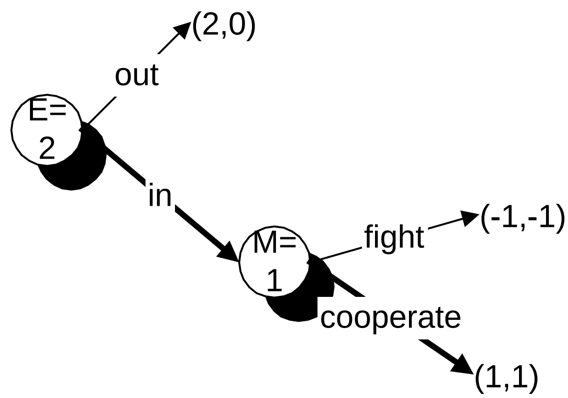
Answers to Problem Set 4: Dynamic Game Theory

May 13, 2002

1. Long Run versus Short Run



subgame perfect equilibrium as marked

	out	in
fight	2*,0*	-1,-1
cooperate	2*,0	1*,1*

Out/fight is Nash, but isn't plausible because the incumbent wouldn't really fight.

Enter/cooperate is subgame perfect in the infinitely repeated game because it is subgame perfect in the stage game.

For the "out" equilibrium in the repeated game, note that after a failure to fight, the equilibrium is the subgame perfect "enter/cooperate" equilibrium. We must find the value of δ for which it is actually optimal for the incumbent to fight if there is entry. (Obviously if he does so, the entrants won't wish to enter.) That is

$$(1-\delta)(-1)+\delta 2 \geq 1$$
$$3\delta \geq 2$$
$$\delta \geq 2/3$$

Unlike the non-perfect equilibrium of the stage game, this makes sense, since the incumbent is actually willing to fight, when the penalty is entry forever afterwards when he does not.

2. Bayes Law

Let E be the evidence and let H be the event that the husband did it.

pr(H)=.8; pr(E|H)=.8; pr(E|~H)=.15

apply Bayes law

$$pr(H|E)=\frac{pr(E|H)pr(H)}{pr(E)}=\frac{pr(E|H)pr(H)}{pr(E|H)pr(H)+pr(E|\sim H)pr(\sim H)}$$
$$=\frac{.8\times .8}{.8\times .8+.15\times .20}=.96$$

so a 96% chance the husband did it. In the second case

$$pr(H|E)=\frac{.8\times .8}{.8\times .8+.05\times .20}=.98$$

3. Mixed Strategy Equilibrium

a) D and R are strictly dominant strategies, so this is the only Nash equilibrium.

b)

	L	R
U	3*,2*	0,0
D	0,0	2*,3*

Two pure equilibria as marked. To the symmetric mixed equilibrium let *p* be the probability L. Then for player 1 to be indifferent, player 2 must mix according to

$3p = 2(1-p)$ giving p=2/5 chance of Land a 3/5 chance of R. For player 2 to be indifferent let q be the chance of D ; we find that q=2/5 as well.

c)

	L	R
U	4*,2	3,5*
D	2,4*	4*,2

2. No pure equilibrium. To find the mixed equilibrium, again, let p be probability of L and q be the probability of D. Then $4p+3(1-p)=2p+4(1-p)$ and $4q+2(1-q)=2q+5(1-q)$ so p=1/3 and q=3/5