Consumption Lock-In

Utility of the stand-in consumer is $\sum_{t=1}^{\infty} \delta^{t-1} \log c_t$ where c_t is annual consumption. Let w_t denote wealth and suppose that there are two assets, stocks and bonds. If the fraction of the portfolio held in stocks is θ_t then

$$w_{t+1} = w_t \theta_t R_t + w_t (1 - \theta_t) - c_t$$

where R_t is the gross rate of return on stocks, a random variable, and we take the gross rate of return on bonds to be 1 (interest rate 0). The key here is that c_t must be chosen before R_t is known.

1. Show that for any distribution of R_t that places positive probability on every neighborhood of zero (for example, the exponential, log-normal, etc.) it is optimal to choose $\theta_t < 0$.

2. Suppose that R_t are i.i.d. binomial (take on just two values). Show that $\theta_t = \theta$ a constant independent of w_t, and calculate what it is.

3. For the Shiller take the mean rate of return on stocks to be the equity premium of 5.6% and the standard error of real stock returns to be 18.1%. Calibrate the corresponding binomial and find the numerical solution to #2 as a function of the subjective discount factor δ. Can you find values of δ for which consumption grows at 1.8% a year (the historical average for per capital consumption)?