Codes of Conduct, Private Information, and Repeated Games

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Motivation

- Self-referential games, Levine and Pesendorfer (Games Econ. Behav., 2007): chance of fathoming others' intention
    - Poker game
    - Skilled interrogation
- Complementarity between repetition and signals
- Folk theorem with private information
    - Issue of coordinating punishments
    - Approximate equilibria: relax exact optimization

Main results

- Generalize Levine and Pesendorfer (Games Econ. Behav.,2007) self-referential game theory
    - Notion of similarity in strategies
    - Model more than two players with multiple roles
- Folk-like theorems with perfect information
- Sustain approximate equilibria
- Strengthen result of Fudenberg and Levine (J. Econ. Theory, 1991) – Approximate folk theorem with private information
    - Sustain ε-Nash equilibria as strict Nash equilibria with self-referentiality

Base Game

- N players *base* game, $i \in \{1, \ldots, N\}$
- Each player i chooses a strategy s_i from a finite set S_i – profile of strategies $s \in S$
    - We allow mixed strategies
- Utility of player i, $u_i(s)$

Self-referential Game

- Players choose codes of conduct from finite space R_0 with profile $r \in R$
- From the finite set of signals Y_i each player observes a signal - denote profile $y \in Y$
- Codes of conduct play two roles:
  - Influence probability distribution $\pi\left(y|r\right)$
  - Each $r^i \in R_0$ induces a map $r^i_j : Y_j \rightarrow S_j$ for all players j - how players play
- Expected utility of player i

$$U_i\left(r\right) = \sum_{y \in Y} \pi\left(y|r\right) u_i\left(r^1_1\left(y_1\right), \ldots, r^N_N\left(y_N\right)\right)$$

Example: Prisoner's Dilemma

- Two players with common set of strategies $S = \{C, D\}$ - Cooperate (C) and Defect (D)
- Space of signals $Y = \{0, 1\}$
- Signals are independent

$$\pi\left(y|r\right) = \pi_0\left(y_1|r\right)\pi_0\left(y_2|r\right)$$

where

$$\pi_0\left(y_i = 1|r\right) = p \qquad \text{if } r^1 = r^2, \text{ and}$$
$$\pi_0\left(y_i = 1|r\right) = q \geq p \qquad \text{if } r^1 \neq r^2$$

Example: Prisoner's Dilemma

- Normal form of the game

	C	D
C	5, 5	0, 6
D	6, 0	1, 1

- One possible equilibrium, static NE – ignoring the signals.
- (Self-referential) Code-of-conduct says $\hat{r}^i = \begin{cases} C & \text{if } y_i = 0 \\ D & \text{if } y_i = 1 \end{cases}$
- Following $\hat{r}$ gives a payoff $5 - 4p$.
- Optimal deviation to $\hat{r}$ is "always defect:" $6 - 5q$

$$\Rightarrow \hat{r} \text{ is preferred only if } q > \frac{1}{5} + \frac{4}{5}p.$$

Example: Two-period Prisoner's Dilemma

- Complements: code-of-conduct and repetition
- Can sustain cooperation even if $q < \frac{1}{5} + \frac{4}{5}p$? i.e. not possible one-shot case
- Let $p = 0 \Rightarrow \pi_0 \left(y_i = 0 | r\right) = 1 \qquad \text{if } r^1 = r^2$
- Code of conduct, $\hat{r}$

	$T = 1$	$T = 2$
if $y_i = 1$	D	D
if $y_i = 0$	C	$tit - for - tat$

- Follow $\hat{r}$ gives expected payoff: 10

Example: Two-period Prisoner's Dilemma

- Deviations:

	C	D
C	5, 5	0, 6
D	6, 0	1, 1

(a) DD :

Expected payoff	$T = 1$	$T = 2$
$7 - 5q$	$q1 + (1-q)\,6$	1

(b) CD :

Expected payoff	$T = 1$	$T = 2$
$11 - 10q$	$(1-q)\,5 + q0$	$(1-q)\,6 + q1$

Complementarity Between Repetition and Self-referentiality

- One-shot game we need $q > \frac{1}{5}$ to adhere to the code-of-conduct and sustain cooperation
- Two-period game: choose $\hat{r} > CD$ only if $q > \frac{1}{10}$ but with $q < \frac{1}{5}$
- With repetition we require a lower probability of detection

From Two Players to Many

- Two players symmetric game
  - Straightforward notion of similarity
- Many players with multiple roles
  - Codes of conduct allow us to extend the previous notion
  - "Be the same:" agree how we would behave, and also how third parties would behave
- Example (Citizens and Politicians)

Perfect Information

- Perfectly revealing signals
- Static two player game, $s \in S$
- $\exists\ y_j^c \in Y_j$ such that $\pi_j(y_j^c|r) = 1$ if $r^1 = r^2$, and $\pi_j(y_j^c|r) = 0$ otherwise.
- Let $\tilde{s}_j^i$ be the (possibly mixed) minmax strategy against player i and $\tilde{u}_i$ be the associated payoff

Theorem (5.1)

For any $v_i = u_i(s_1, s_2) \geq \tilde{u}_i$ for all $i = 1, 2$ and $(s_1, s_2) \in S$, there exists a profile of codes of conduct r such that (v_1, v_2) is a Nash self-referential equilibrium payoff.

Self-referential Punishment

- Base game: Each player has access to N randomizing devices each of which has independent probability $\varepsilon_R > 0$ of event *punishment*
- Y, complete R and $\pi(y|r)$

Definition

The self-referential game is said to **E,D permit detection** where $1 \geq E, D \geq 0, E + D \leq 1$ if for every player i there exists a player j and a set $\overline{Y_j} \subset Y_j$ such that for any code of conduct $r \in R$, any signal $\bar{y}_j \in \bar{Y}_j$, and any $\tilde{r}^i \neq r^i$ we have $\pi_j\left(\bar{y}_j|\tilde{r}^i, r^{-i}\right) - \pi_j\left(\bar{y}_j|r\right) \geq D$ and $\pi_j\left(\bar{y}_j|r\right) \leq E$.

- D probability of detection
- E probability of false positive

Strategies in the Approximate Equilibria

- Given strategies $s^0, s, \{s_j^i\}$
- s^0 is an ε_0-Nash equilibrium of base game
- $s_{(j)}^i = \left(s_j^i, s_{-j}\right)$ are ε_1-Nash equilibrium satisfying for all i we define $\underline{P}$ – a lower bound for the size of the punishment

$$P_i = u_i\left(s^0\right) - u_i(s_{(j)}^i) \geq \underline{P} \geq 0$$

and for some $\varepsilon_p \geq 0$

$$\left| u_j(s_{(j)}^i) - u_j\left(s^0\right) \right| \leq \varepsilon_p$$

Parameters

- Highest and lowest payoff $\overline{u}, \underline{u}$
- We define

$$\varepsilon = \varepsilon_0 + (N + \overline{u} - \underline{u})(\varepsilon_1 + \varepsilon_p) E$$

$$K = \max\left\{(N + \overline{u} - \underline{u})\left[3N^2(1 + \overline{u} - \underline{u})\right], \left[N^5(\overline{u} - \underline{u}) + N\right](\overline{u} - \underline{u})\right\}$$

- Depend on number of players, highest and lowest payoffs

Sustaining Approximate Equilibria

- Small probability of detecting deviations from a code-of-conduct can be used to sustain approximate equilibria of the base game as strict equilibria of the self-referential game.

Theorem (7.1)

Suppose $(D(\underline{P}-\varepsilon_1))^2 > 4K\varepsilon$. Then there exist an ε_R and a strict Nash equilibrium code-of-conduct r with

$$\left|u_i\left(s^0\right)-u_i\left(r\right)\right| \leq \varepsilon + D\left(\underline{P}-\varepsilon_1\right) - \sqrt{\left(D\left(\underline{P}-\varepsilon_1\right)\right)^2 - 4K\varepsilon}, \qquad \text{for all } i$$

Repeated Self-referential Games with Private Information

- Class of base games, repeated games between patient players: Rich structure of approximate equilibrium
- E – how frequently we punish on the equilibrium path if nobody deviates – is fixed and not necessarily small
- Fudenberg and Levine (J. Econ. Theory, 1991) show that socially feasible payoff vectors that Pareto dominate mutual threat points are ε-sequential equilibria where $\varepsilon \longrightarrow 0$ as $\delta \longrightarrow 1$.

Folk Theorem

- The following result is a discounted strict Nash folk theorem for enforceable mutually punishable payoffs in repeated self-referential game with private information:

Theorem (8.3)

If V^* has no empty interior, if the game is informationally connected, if for some $E \geq 0, D > 0$ the self-referential T discrete versions E, D strongly permits detection, and if $v \in V^*$ then there exists a sequence of discount factors $\delta_n \to 1$, discretizations T_n and codes of conduct r_n such that r_n is a strict Nash equilibrium for δ_n, T_n, and $u_i(r_n; \delta_n, T_n) \to v_i$.

Folk Theorem

- It suffices to prove then next result by Theorem 7.1:

Theorem (8.4)

If V^* has no empty interior, if the game is informationally connected, if for some $E \geq 0, D > 0$ the self-referential T discrete versions E, D permits detection, and if $v \in int\left(V^*\right)$ then for any $\varepsilon_0 > 0$, there exists a discount factor δ, a discretization T and strategy pairs s_i^0, s_i^j such that s^0 is an ε_0-Nash equilibrium for $\delta, T, \varepsilon_1 = \varepsilon_p = \varepsilon_0$ and $\underline{P} = \sqrt[3]{\varepsilon_0}$.

Conclusion and Final Remarks

1. Relevance of self-referential game theory
   - Understanding opponent's intentions
   - Two roles: Both generate and respond to signals
2. Folk-like theorems with perfect information
   - Application: Sustain cooperation in prisoner's dilemma game
3. For a given approximate equilibrium of the base game we can find a strict Nash equilibrium of the self-referential game using code-of-conduct
4. We proved a folk theorem in repeated games with private information
   - For approximate equilibria we strengthen the result of ε-Nash to strict Nash equilibrium