Political Economy Problems

1. There is a government that moves first and commits to a taxation policy: the probability with which capital is taxed τ. There are a finite number N of households who move in response to the taxation policy. (The actual taxes are decided after investment decisions are made.) Here you are asked to examine the Stackelberg equilibria of the game. Households are endowed with a unit of capital and choose the amount $x \leq 1$ to invest. If capital is not taxed it carries a rate of return $r > 0$, if it is taxed it carries no return. If households invest the maximum and capital is taxed the government receives a revenue of $1 + r$ per household If capital is not taxed the government raises the same amount of revenue by means of a distortionary labor tax costing a representative household $c > 1 + r$. The utility for a household is $(1 - \tau)(1 + rx - c) + \tau(1 - x)$. The government gets the same utility as households, except that if it taxes capital and households invest less than the full amount there is a revenue shortfall resulting in a loss to the government (per household) of $p(1 - x)$ where $p > 1$. Hence government utility is $(1 - \tau)(1 + rx - c) + \tau(1 - x)(1 - p)$.

a. Show that the first best is $\tau, x = 1$.

b. The Ramsey equilibrium or second-best is the Stackelberg equilibrium where the government commits to a fixed probability of taxes τ and the most favorable incentive compatible decision by households occurs. Show that this is given by $\tau = 1/(1 + r), x = 1$.

Now suppose that each individual generates an independent noisy signal y of the amount invested x and that the government observes the average value of this signal $\overline{y}$ and can commit to a policy of choosing how much to tax as a function of $\overline{y}$.

c. Show that if there is no noise in the signal, so that $y = x$ then there is a policy for the government that yields the first best.

Now suppose that the signal is normally distributed with mean x and variance $\sigma^2 > 0$. The government may choose a threshold $\hat{y}$ such that it taxes capital with probability one if and only if $\overline{y} > \hat{y}$, otherwise it does not tax capital.

d. Find the optimal value of $\hat{y}$.

e. What happens as $N \to \infty$?

2. Suppose there are two groups $k = 1, 2$ who are competing for a prize worth V_k to group k. One way to model lobbying or voting effort is to imagine that the groups engage in continuing expenditure in the lead up to the election or politician decision. The key point is that each can see whether the other is spending and adjust its own spending accordingly. A simple model of this is the war of attrition game. Here each group may spend up to an amount c per unit time. Think of this as spending on political organization. If spending drops below c at some point in time, the political organization is disbanded or goes bankrupt effectively conceding the decision to the other group. Hence for each group the question: how long should I stay in? Show that this game has three equilibria, two pure and one mixed, and find the mixed equilibrium.

3. Suppose voters types y are uniformly distributed on $[0,1]$ and that the cost of voting is an increasing function $c(y)$. Suppose that a threshold $\hat{y}$ is chosen with voters with lower costs to vote and voters with higher costs not to vote. Anyone who votes is not punished. Anyone who does not vote generates a signal π_1 if $y < \hat{y}$ and with probability $\pi_2 \leq \pi_1$ if $y > \hat{y}$. The greatest possible punishment is $\overline{P}$. Suppose there are two levels of punishment chosen: P_1 if the signal is received and P_2 if it is not received (recalling that in both cases the individual did not vote).

a. Characterize incentive compatible pairs $0 \leq P_1, P_2 \leq \overline{P}$.

b. Find the incentive compatible pair that minimizes the expected cost of punishment when voters follow the social norm.

c. What is the least punishment cost?