Economics Letters 25 (1987) 301–302
North-Holland

NASH EQUILIBRIA EQUAL COMPETITIVE EQUILIBRIA

David K. LEVINE *

University of California, Los Angeles, CA 90024, USA

Received 30 July 1987
Accepted 15 September 1987

With strictly monotone preferences and a continuum of traders, there is a game in which the set of Nash equilibria is exactly the same as the set of competitive equilibria.

1. Introduction

With a continuum of traders it is easy to construct games in which Nash equilibria and competitive equilibria overlap. For example, everyone announces a desired net trade; if the announcements turn out to be a competitive equilibrium everyone trades the announced amount; otherwise, no one can trade. Of course, no trade at all is also a Nash equilibrium. If the game must be defined independent of preferences, have everyone announce his utility function. Calculate a competitive equilibrium with respect to the announcements in such a way that prices do not change if a measure zero set of individuals deviates, and assign the corresponding competitive allocation. Telling the truth is weakly dominant, and therefore a Nash equilibrium. However, if there are multiple competitive equilibria, only the chosen one is a Nash equilibrium. This latter game has been used in the study of trading mechanisms, for example, by Hammond (1979), and Makowski and Ostroy (1986).

Here we show that if preferences are strictly monotone, there is a game in which the sets of Nash and competitive equilibria are exactly the same. Moreover, the game is defined independent of individual preferences, and each individual has a finite dimensional strategy space.

2. The model

We model a continuum of consumers $t \in [0,1]$. The commodity space is $\mathbb{R}^m_+$, and an *allocation* is a measurable map from the unit interval to $\mathbb{R}^m_+$, written x_t. The endowment is an allocation ω_t with $\int \omega_t \mathrm{d}t < \infty$, while preferences are given by strictly monotone utility functions u_t: $\mathbb{R}^m_+ \to \mathbb{R}$. A *competitive equilibrium* is a price vector π^* on the unit simplex and an allocation x_t^* that is socially feasible, that is, $\int (x_t^* - \omega_t)\,\mathrm{d}t \leqslant 0$, and individually rational, that is, $u_t(x_t^*) = \max_{\pi^*(x_t - \omega_t) \leqslant 0} u_t(x_t)$, for almost all t.

* I would like to thank Lloyd Shapley and William Zame. This research was sponsored by NSF grant 85-09484 and the UCLA Academic Senate.

A game that implements the competitive equilibria as Nash equilibria defines a *strategy* for t to be a desired consumption vector $x_t \in \mathbb{R}_+^m$ and a price vector p_t on the unit simplex that jointly satisfy $p_t(x_t - \omega_t) \leqslant 0$. Let S be the space of strategies (x_t, p_t) that are measurable with respect to t. An outcome function is a map y_t: $S \to \mathbb{R}_+^m$ that is socially feasible, so satisfies $\int (y_t - \omega_t)\,dt \leqslant 0$. If $s^* \in S$, let $s^* \backslash s_t$ be the element of S given by s_τ^* for $\tau \neq t$ and s_t for t. A *Nash equilibrium* with respect to y is an $s^* \in S$ such that

$$u_t(y_t(s^*)) \geqslant u_t(y_t(s^* \backslash s_t))$$

for all strategies s_t and almost all t.

Let $(x, p) \in S$ be given: we show how to construct $y(x, p)$ so that Nash equilibria correspond one to one with competitive equilibria. Since p is measurable, if π is on the unit simplex, $p^{-1}(\pi)$ is a measurable set, and since x is measurable, $\int_{p^{-1}(\pi)} (x_t - \omega_t)dt$ is well-defined (although possibly equal to $+\infty$).

For each $\epsilon > 0$ there are finitely many values of π for which the disjoint sets $p^{-1}(\pi)$ have measure ϵ or larger. Consequently, there are countably many values of π for which $p^{-1}(\pi)$ has positive measure, and at least one (in fact uncountably many) value of π for which $p^{-1}(\pi)$ has measure zero. For such a π $\int_{p^{-1}(\pi)}(x_t - \omega_t)\,dt = 0$. For each equivalence class of $(x, p) \in S$, that differ only on a set of measure zero, we can therefore pick a single $\pi(x, p)$ according to the rules:

(1) if $\int(x_t - \omega_t)\,dt \leqslant 0$ and for some π $p^{-1}(\pi)$ has full measure, pick π,

(2) otherwise pick any π for which $\int_{p^{-1}(\pi)}(x_t - \omega_t)\,dt \leqslant 0$.

Moreover, we can assign y_t to be ω_t if $p_t \neq \pi(x, p)$ and $y_t = x_t$ if $p_t = \pi(x, p)$. This is a game by our definition.

3. The theorem

If x_t^*, π^* is a competitive equilibrium, consider the point x_t^*, $p_t^* \equiv \pi^*$ in S. For almost all t, $u_t(x_t^*) = \max_{\pi^*(x_t - \omega_t) \leqslant 0} u_t(x_t)$. For such a t, deviating to $p_t \neq \pi^*$ is equivalent to choosing $p_t = \pi^*$ and $x_t = \omega_t$. Given that he has chosen $p_t = \pi^*$, t is restricted to plans x_t with $\pi^*(x_t - \omega_t) \leqslant 0$ by the definition of a strategy; moreover, the utility received is $u_t(x_t)$. It follows that x_t^* is at least as good as any other feasible choice. Consequently every competitive equilibrium is a Nash equilibrium.

If x_t^*, p_t^* is a Nash equilibrium, let $y_t^* = y_t(x^*, p^*)$ be the final allocation, and let π^* be the price chosen according to the rules above. By definition of y, $\int(y_t^* - \omega_t)\,dt \leqslant 0$. It is also feasible for t to deviate to $p_t = \pi^*$ and any $x_t \in \mathbb{R}_+^m$ with $\pi^*(x_t - \omega_t) \leqslant 0$. For almost all t, then,

$$u_t(y_t^*) \geqslant \max_{\pi^*(x_t - \omega_t) \leqslant 0} u_t(x_t).$$

Since utility is strictly monotone, this implies $\pi^*(y_t^* - \omega_t) \geqslant 0$. Since $\int(y_t^* - \omega_t)\,dt \leqslant 0$, $\int \pi^*(y_t^* - \omega_t)\,dt \leqslant 0$, and it follows that for almost all t $\pi^*(y_t^* - \omega_t) = 0$. This shows that every Nash equilibrium is a competitive equilibrium.

References

Hammond, P., 1979, Straightforward individual incentive compatibility in large economies, Review of Economic Studies 46, 263–287.

Makowski, L. and J. Ostroy, 1986, Dominant strategy mechanisms in nonatomic transferable utility economies, Working paper no. 421 (University of California, Los Angeles, CA).