Time, Interest, and Discounting the Future

interest at an annual rate of *r*

paid annually:

\$1 in the bank, and in one year collect \$1+r

discount factor:

to have \$1 in the bank in one year's time, must put

$\delta = \dfrac{1}{1+r}$ in the bank today

A Useful Approximation

$$\frac{1}{1+r} \approx 1 - r \text{ if } r << 1$$

r	$\frac{1}{1+r}$	$1 - r$
1%	.9901	.9900
10%	.9091	.9000
50%	.6667	.5000

Present Value

1 dollar at the beginning of every year for τ years is worth what right now?

what is $z = 1 + \delta + \delta^2 + ... + \delta^{\tau-1}$?

$\delta z = z - 1 + \delta^\tau$

$(1 - \delta)z = 1 - \delta^\tau$

$$\boxed{1 + \delta + \delta^2 + ... + \delta^{\tau-1} = z = \frac{1 - \delta^\tau}{1 - \delta}}$$

***the one to remember** $1 + \delta + \delta^2 + ... = 1/(1 - \delta)$

**subtract the tail

Mortgage Interest

You buy a house for \$250,000. You make a 20% down payment, and get a 30 year fixed rate mortgage at 8% annual interest. How much are your monthly payments.

- suppose that monthly interest is 8%/12=0.67%
- so $\delta = \dfrac{1}{1+.0067} \approx .9933$
- mortgage is for \$200,000
- number of payments $\tau = 360$

Note: what is δ^τ? 0.08

Find the Monthly Payment

let *p* be the monthly payment

then

$$200000 = (\delta + \delta^2 + \ldots + \delta^\tau)p = \delta \frac{1-\delta^\tau}{1-\delta} p$$

*shift left and multiply by δ

or

$$p = 200000 \frac{1}{\delta} \frac{1-\delta}{1-\delta^\tau}$$
$$\approx 200000 \frac{1}{.9933} \frac{.006655}{.9111} \approx 1471$$

Capital and Investment

Present value is an essential tool for evaluating investments

Investment creates *capital*

there are several types of capital

- financial capital (for example, a bank account)
- physical capital (a house, machine, factory)
- human capital (the extra future income you get by investing in education)

Next: **Repeated Games**

The Repeated Prisoner's Dilemma

	Player 2	
Player 1	**don't confess**	**confess**
don't confess	32,32	28,35
confess	35,28	30,30

- This is a simultaneous move game with a unique Nash equilibrium, and a unique strictly dominant strategy solution at 30, 30.
- The unique non-cooperative solution is Pareto dominated by 32, 32
- with repeated play, incentives are changed by the possibility of punishments and rewards in the future.

More Than One Equilibrium

a basic feature of repeated games: regardless of the discount factors, the repeated static equilibrium is a subgame perfect equilibrium of the repeated game

Grim Trigger Strategies

the *grim trigger strategy* in the repeated game is

- cooperate in the first period
- cooperate in subsequent periods as long as all players have cooperated in every previous period
- cheat in any period in which some player has cheated in any previous period

What to Do?

- payoff to cheating

$$\left(35 + 30\delta + 30\delta^2 \ldots\right) = 5 + 30/(1-\delta)$$

- payoff to cooperating

  $32/(1-\delta)$

- optimal to cooperate if

  $32 \geq 35 - 5\delta$ or

  $\delta \geq 3/5$

- if $\delta \geq 3/5$ both players playing the grim strategy is a subgame perfect equilibrium
- why is this subgame perfect?

Incentive Compatibility

The condition that cooperation is better than cheating

$$32 \geq 35 - 5\delta$$

is called an *incentive constraint*

if it is satisfied then cooperation is said to be *incentive compatible*

Pedro Dal Bo' s Experiment

PD1					PD2	
		Blue Player			Blue Player	
		C	D		C	D
Red Player	C	65 , 65	10 , 100		75 , 75	10 , 100
	D	100 , 10	35 , 35		100 , 10	45 , 45

All payoffs in the game were in points. At the end of each session, the points earned by each subject were converted into dollars at the exchange rate 200 points=$1 and paid privately in cash. In addition, subjects were paid a 5 dollar show up fee

Rotating matching

Repetition

Infinite horizon

$\delta = 0, 1/2, 3/4$ expected length $1, 2, 4$ **(how did he do this??)**

Finite horizon

$H = 1, 2, 4$

subjects played all infinite or all finite

done in both orders – increasing length and decreasing length

Theory

δ	PD1	PD2
0	DD	DD
½	DD, DC, CD	DD, CC
¾	All	All

Results on Cooperation

Table 5: Percentage of cooperation by match and treatment*

		Match									
		1	2	3	4	5	6	7	8	9	10
	$\delta = 0$	26.26	18.18	10.61	11.62	12.63	12.63	5.56	5.26	5.26	5
Dice	$\delta = \frac{1}{2}$	28.36	27.12	34.58	35.53	21.60	19.08	29.84	35.96	28.16	50
	$\delta = \frac{3}{4}$	40.44	28.57	27.78	32.92	46.51	33.09	44.05	53.51	42.26	45.83
	$H = 1$	26.56	18.23	16.67	17.19	11.98	8.02	6.79	10.49	6.14	6.67
Finite	$H = 2$	19.79	15.89	14.84	9.64	11.46	10.80	12.04	10.19	6.58	6.67
	$H = 4$	31.64	30.34	30.47	25.52	25.13	23.77	16.36	19.75	14.91	20.83

*All rounds.

Focus on matches 4-10

Table 6: Percentage of cooperation by round and treatment*

		Round											
		1	2	3	4	5	6	7	8	9	10	11	12
	$\delta = 0$	9.17											
Dice	$\delta = \frac{1}{2}$	30.93	26.10	19.87	12.50	12.96							
	$\delta = \frac{3}{4}$	46.20	40.76	38.76	34.58	33.04	27.27	24.75	26.28	29.17	26.04	32.29	31.25
	$H = 1$	10.34											
Finite	$H = 2$	13.31	6.90										
	$H = 4$	34.58	21.55	18.97	10.63								

*Matches four through ten.

Joint Outcomes

Table 7: Distribution of outcomes by stage game and treatment*

	$\delta = 0$		$\delta = \frac{1}{2}$		$\delta = \frac{3}{4}$	
	PD1	PD2	PD1	PD2	PD1	PD2
CC	2.98	0.27	**3.17**	**18.83**	20.68	25.64
CD & DC	20.83	13.98	**28.57**	**25.50**	30.34	26.03
DD	76.19	85.75	68.25	55.67	48.98	48.33

*Matches four through ten, and all rounds.

Crime, Punishment and the Future

- Why do you pay your taxes?
- Why do soldiers fight?
- Why do you repay your debts?

For fear of future punishment.

- The simple repeated PD game is a paradigm of how a society functions
- In the grim trigger strategy cooperation arises because of failure to cooperate leads to future punishment

The idea continues to apply in more “realistic” environments

- limited information, many people, different payoff possiblities

Concepts

- **interest rate**, **discount factor**
- **present value**
- investment, capital
- financial capital, physical capital, human capital
- grim trigger strategy
- **incentive compatibility**

Skill

given investments with different income streams

find and compare the present values

determine when the incentive constraints are satisfied by grim-trigger strategies